

Optical levitation-associated atomic loading in a dipole trap

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Abstract

We demonstrate a robust method for the efficient optically levitated loading of ultracold Cs atoms in a crossed dipole trap. When preparing a large number of atomic samples, a large-volume crossed dipole trap is required to form a shallow but very efficient loading potential. The scattering force coming from a red near-off-resonance laser is utilized to precisely compensate for the destructive gravitational force of the atoms, making it a promising method for loading and trapping $\sim 5.1 \times 10^6$ atoms in a pure optical trap. The optimum levitation laser intensity is $\sim 177 \text{ W cm}^{-2}$ with a detuning of -25.0 GHz . The dependence of the variation in the number of atoms loaded and trapped in the optically levitated dipole trap on the intensity and detuning of the levitated laser is in good agreement with theoretical predictions.

Keywords: optical levitation, dipole trap, scattering force

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, optical dipole traps have become a central tool in the physics of cold atoms, since they allow for great versatility in trapping potentials and offer the possibility of studying and implementing numerous physical situations, such as double wells [1, 2], microscopic traps [3, 4] and optical lattices [5, 6]. Loading as many atoms as possible into the dipole trap is important in many fields of modern physics involving these optically trapped cold atoms, and is especially relevant to future applications in quantum computation [7, 8] and the exploration of novel many-particle quantum effects [9, 10]. For some special atomic species, such as Cs, preparing a large number of atoms in an optical trap is very important

for further research, and it should be possible to load and trap cold atoms in a large-volume crossed dipole trap [11]. Due to the limitations of laser power, large-volume dipole traps formed by laser beams with a large waist are often 'shallow'. Affected by the anti-trapping potential induced by the gravity of atoms, magnetic levitation-associated loading [12–14] has been successfully used to achieve an available trapping potential and efficient loading. While the preparation and manipulation of atoms with a magnetic field also has some limitations, a new method known as optical levitation could be used to load atoms into a dipole trap.

Optical levitation by radiation pressure is widely applied to offset the gravity in the divergent region of a focused laser beam, as was first demonstrated by Ashkin in 1970 [15, 16]. Up to now, there have been extensive studies and promising applications relating to optical levitation, such as three-dimensional force-field microscopy [17], levitating granules [18, 19], dual-beam fibre-optic traps [15], optical tweezers [20], and so on. When loading atoms into an optically levitated



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dipole trap, the levitated laser provides an axial force lifting the atoms to an equilibrium position where all force components are balanced.

Compared with pure optical dipole traps, magnetic levitation-associated loading often cannot be applied to systems that do not have a permanent magnetic moment, such as Yb [21] and alkaline-earth atoms [22]. Moreover, control of the magnetic field is unfeasible when the induced electromotive force is non-negligible for changing the magnetic field in a fast manner. All of these problems can be avoided if the atoms are loaded in a pure optical dipole trap without a magnetic field. The laser intensity and pattern can be turned on a fixed length scale rapidly in time, which could be used in many applications [23]. This is one motivation for the development of various optical dipole traps, and opens the possibility of the efficient loading of ultracold atoms and molecules in arbitrary spin states. It is well known that a near-off-resonance levitated laser will heat the atoms and induce a huge loss of atoms. However, the method can be used for precision measurement of photoassociation spectroscopy for the rapid control of the laser. Furthermore, this method is promising for studies on ultracold atoms and molecules in nanofibres, where an ultra-low temperature (\sim nK) of the atomic sample is not usually required. If a random spatial intensity distribution of the laser is applied, the scattering length of the atomic samples will vary in different positions, which might induce the production of new atomic quantum phases [23]; all of which cannot be achieved with a magnetic field.

In this paper, we demonstrate the optical levitation loading of cold Cs atoms in a large-volume crossed dipole trap. Our research is based on a relatively simple experimental setup. A near-off-resonant laser is used to compensate for the gravitational force, and the dependence of the number of trapped atoms on the detuning and the intensity of the laser are measured. A simple theoretical model is provided to explain the experimental results. In addition, we observe a controlled local loading of an atomic sample in an optical trap.

2. Theoretical model

Before presenting our results, we give a brief theoretical description of the optical potential. The upward scattering force on the atoms induced by the levitation laser is given [24] by

$$F_{scatt} = \frac{\hbar\pi\Gamma}{\lambda} \frac{I/I_S}{1 + I/I_S + 4(\Delta/\Gamma)^2} \quad (1)$$

where Γ is the natural linewidth of the excited Cs atoms in the $6P_{3/2}$ state, I_S is the saturation intensity of the Cs atoms, λ is the wavelength, I is the intensity of the levitation laser, and Δ is the frequency detuning from the atomic transition $6S_{1/2}$, $F = 3 \rightarrow 6P_{3/2}$, $F' = 4$. Considering the gravity of the atoms and the effect of the levitation laser, the total optical potential in a dipole trap [14] is

$$U(x, y, z) = -\frac{3c^3}{\omega_0^3} \left(\frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right) S + (F_{scatt} - mg)z \quad (2)$$

where $S = \frac{P_1}{\omega^2(x)} e^{-2(y^2+z^2)/\omega^2(x)} + \frac{P_2}{\omega^2(y)} e^{-2(x^2+z^2)/\omega^2(y)}$, (x, y, z) are the Cartesian spatial coordinates of the atom with the z -axis oriented downwards, c is the speed of light, $\omega = 2\pi c/\lambda$ is the circular frequency of the dipole trap laser, ω_0 is an effective transition circular frequency defined by a weighted average of both D lines for the Cs atoms, P_1 and P_2 are the powers of the two beams of the dipole trap laser, and $\omega(x)$ and $\omega(y)$ are the corresponding beam waists. The optical potential in the horizontal position is determined by the two crossed dipole laser beams. The potential induced by the conservative force of the levitated laser in the vertical direction is less than $1 \mu\text{K}$, which has a negligible influence on the loading effect. The scattering force depends on velocity and is not conservative, so the 'optical pressure' (F_{scatt}) caused by the levitation laser is mainly applied to compensate for the destructive gravity potential.

Using equation (2), we computed the theoretical potentials acting on the Cs atoms in the vertical direction, and the results are shown in figure 1. The solid red line represents the potential of the dipole trap alone (no gravity and no levitation force), while the solid black line represents the total potential.

The vertical optical potential is determined by the laser intensity and the detuning from the atomic hyperfine resonant transition of the levitation laser. The curves in figure 1 were computed with a detuning of -23.70 GHz and a laser intensity varying from 0 to 197 W cm^{-2} . In figure 1(a), it is clear that the potentials of the crossed dipole trap barely capture the atoms without the levitation laser. Figure 1(b) gives the potential curve for the laser intensity of 108 W cm^{-2} , which indicates that the total trap potential increases, providing effective levitation. Based on the relation $F_{scatt} - mg = 0$, a levitation laser intensity of 169 W cm^{-2} is expected to cancel out the gravity completely, as shown in figure 1(c). A further increase in the laser intensity causes the destructive potential to form again, as shown in figure 1(d).

3. Experimental setup

The experimental setup is shown schematically in figure 2. Samples of cold atoms are produced in a standard vapour-loaded Cs magneto-optical trap (MOT). Following the achievement of a compressed MOT and optical molasses, 3×10^7 atoms are obtained with a peak density of $\sim 10^{11} \text{ cm}^{-3}$. Degenerated Raman sideband cooling (DRSC) is applied to the atomic cloud to cool and polarize the atoms in the desired $F = 3$, $mF = 3$ state. The temperature of the atomic cloud is $\sim 1.7 \mu\text{K}$ [25]. To transfer more cold Cs atoms into a dipole trap, we superimpose a large-volume crossed dipole trap onto the atomic sample released from the optical lattice of the DRSC. For investigation of the optical levitation-associated atomic loading in a dipole trap, all of the magnetic field was switched off after the DRSC, before we loaded atoms into the dipole trap. The dipole trap is produced by a 1070 nm, multifrequency, linearly polarized fibre laser (IPG Photonics) and consists of two horizontal beams crossing at an angle of 90° . In terms of power, the two laser beams of the crossed dipole trap are 7.0 W and 7.2 W , and the corresponding beam waists are $230 \mu\text{m}$ and

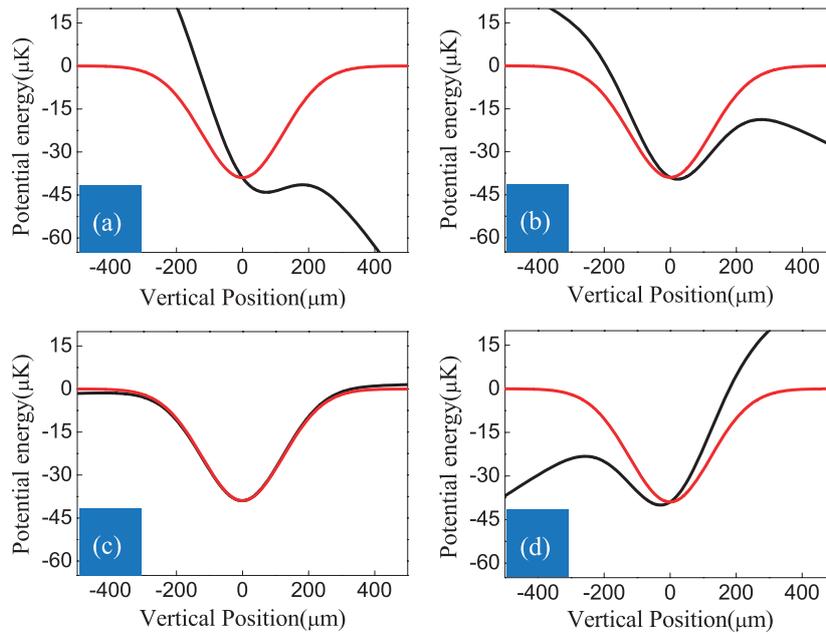


Figure 1. Potentials of an optically levitated crossed dipole trap in the vertical direction, at a levitation laser intensity of: (a) 0 W cm^{-2} ; (b) 108 W cm^{-2} ; (c) 169 W cm^{-2} ; and (d) 197 W cm^{-2} . The detuning of the levitation laser from the atomic hyperfine resonant transition is -23.70 GHz . The solid red line represents the potential of the dipole trap alone (without gravity and levitation force), and the solid black line represents the total potential.

$240 \mu\text{m}$ at the centre of the trap, respectively. Two acousto-optic modulators (AOMs) (Crystal Technology) with 110 MHz frequency shifts are used for intensity stabilization of the two dipole trap laser beams and rapid on/off switching in less than $1 \mu\text{s}$. During the loading process of the crossed dipole trap, we simultaneously switch on a near-off-resonance linearly polarized laser along the vertical (z) direction in order to compensate for the gravity of the Cs atoms without a magnetic field. The levitation laser is provided by a custom-made tunable grating feedback diode laser with a central wavelength of 852 nm . The beam waist is $180 \mu\text{m}$ and the maximum output laser intensity is 226 W cm^{-2} .

We were able to obtain a maximum of $\sim 5.1 \times 10^6$ atoms in the levitated dipole trap, as measured using the absorption image (Apogee CCD camera) taken in the horizontal direction after 3 ms of expansion after release from the levitated dipole trap. The image itself is shown in figure 3(a), and the corresponding distribution of optical intensity is shown in figure 3(b). The optical potential induced by the red-detuned levitation laser is an order of magnitude larger than the potential of the crossed dipole trap. When the laser is blue-detuned from the resonant transition, it will result in a loading rate of almost zero, which has been demonstrated in our experiment. We also measured the number of atoms as a function of the storage time in the dipole trap, as shown in figure 3(c); the inset shows the experimental sequence (we switched the two dipole laser beams and the levitation laser on and off simultaneously). A large loss is observed within the initial 40 ms , and this is mainly attributed to the absorption of photons by atoms and to the effects of light-assisted collisions, which cause a large number of atoms to be heated to a sufficient level to escape the trap with an increase in storage time.

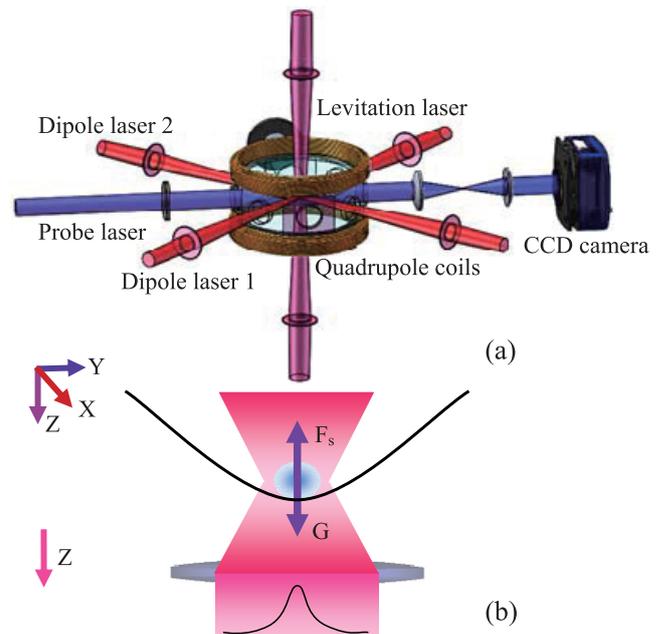


Figure 2. (a) Experimental setup. Dipole lasers 1 and 2 are applied to create the crossed dipole trap, and a levitation laser is used to compensate for the destructive gravitational force of the atoms. The probe laser beam passes through the trapped atoms, and the number of atoms is measured using the absorption image. Quadrupole coils are used to produce the gradient magnetic field for the MOT. (b) Schematic diagram of optical levitation in the vertical direction.

4. Experimental results and discussion

Since the number of atoms loaded into the levitated dipole trap is proportional to the depth of the trap, the levitation effect is reflected by the dependence of the number of atoms

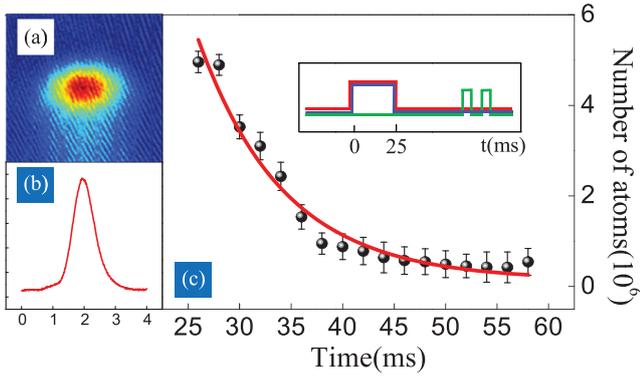


Figure 3. (a) Absorption image taken along the horizontal direction after 40 ms loading of the levitated dipole trap. (b) The corresponding distribution of the optical intensity along the horizontal direction. (c) Number of atoms remaining in the levitated crossed dipole trap as a function of time: the dots (blue) show experimental data; the solid curve (blue) is the fit with the exponential function; and the inset shows the sequence of dipole laser (red), levitation laser (blue) and probe laser (green) pulses.

in the trap on both the detuning and the intensity of the levitation laser. In order to study this dependence, we performed a series of experiments in which we varied the laser intensity and detuning, and detected the number of trapped atoms using an absorption imaging technique by releasing them from the levitated dipole trap after loading.

Figure 4(a) shows the variation in the number of atoms in the trap with the intensity of the levitation laser at a detuning of -23.7 GHz from the resonant transition. With an increase of the levitation laser intensity, the number of atoms in the dipole trap first undergoes an exponential type of growth. The maximum number of atoms held in the dipole trap was measured as 3.4×10^6 when the laser intensity was 177 W cm^{-2} ; the solid red line is a fitting curve by equation (2). Although the loss of atoms was non-negligible, the number of atoms almost corresponded to the depth of the trap. The trapped atoms decrease with a further increase in the laser intensity, as they are pushed from the dipole trap.

In order to study the influence of detuning, the intensity of the levitation laser was fixed at 177 W cm^{-2} while the detuning was varied. The variation of the number of atoms is shown in figure 4(b). With a decrease in the detuning, the number of atoms trapped in the dipole trap first undergoes an exponential-like growth, up to a maximum number of trapped atoms of $\sim 5.1 \times 10^6$ at a detuning of -25.0 GHz. A further decrease in the detuning causes a rapid decrease in the number of trapped atoms. When the frequency of the levitation laser approaches that of the atomic resonance transition, a large loss of trapped atoms is observed.

It is an observable fact that the near-off-resonant levitation laser heats the atomic sample, which also leads to the short lifetime of the atoms in this optically levitated dipole trap. The atomic temperature presents an exponential growth with the time that the atoms remain in the dipole trap increasing (the temperature reaches $\sim 13.7 \mu\text{K}$ in 50 ms). The heating is mainly from the spontaneous scattering of trap photons. For our optically levitated loading technology, the fluctuations of

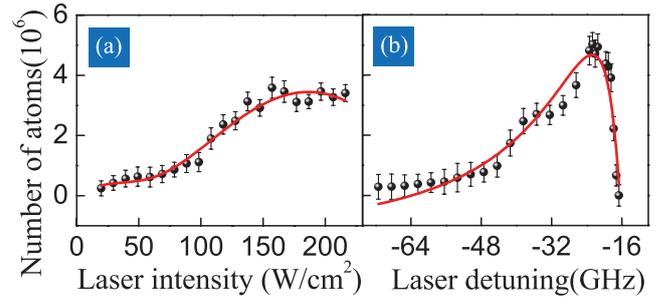


Figure 4. (a), (b) Number of atoms in the optically levitated dipole trap as a function of laser intensity and detuning: the laser detuning in (a) was -23.7 GHz; the laser intensity in (b) was 177 W cm^{-2} .

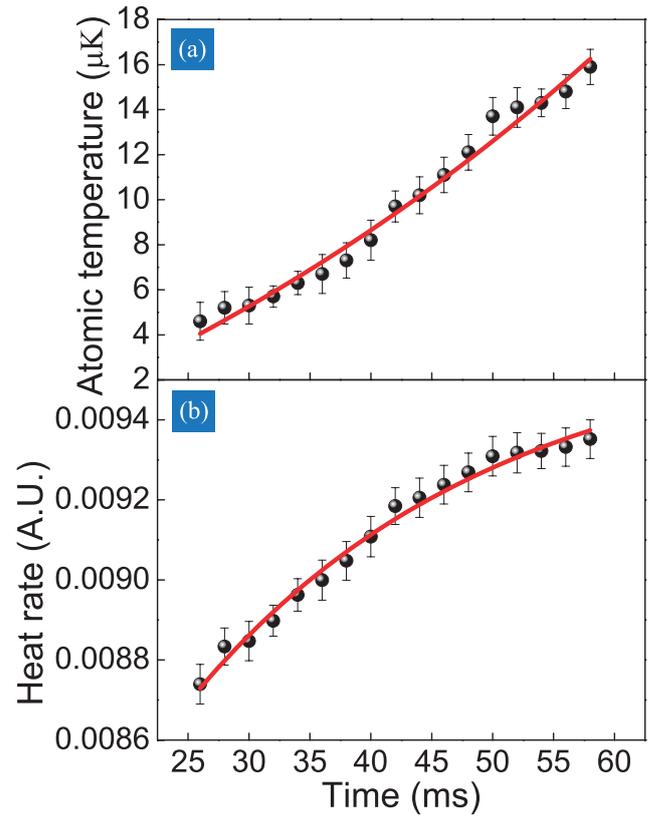


Figure 5. (a), (b) Temperature and heating rate of atoms in the optically levitated dipole trap as a function of time.

the levitated laser radiation force induced huge spontaneous scattering, which heated the atoms rapidly. In our experiment, the detuning of the levitated laser ranges from several GHz to dozens of GHz, the energy of the spontaneous scattering photons is derived from the frequency of the lasers, not of the optical transition [26]. For the purpose of describing the atom heating mechanism, we calculate the heating rate of the atoms.

In a dipole trap, the heating rate can be expressed as

$$\dot{T} = \frac{2/3}{1 + \kappa} T_{rec} \bar{\Gamma}_{sc} \quad (3)$$

where $\bar{\Gamma}_{sc}$ is the mean scattering rate, which can be calculated from the temperature of the sample, U_0 is a constant offset, $\bar{\Gamma}_{sc} = \frac{\Gamma}{\hbar\Delta} (U_0 + \frac{3\kappa}{2} k_B T)$, Δ is the detuning of the levitated laser, $\kappa \equiv \bar{E}_{pot}/\bar{E}_{kin}$ is the ratio between the potential and the

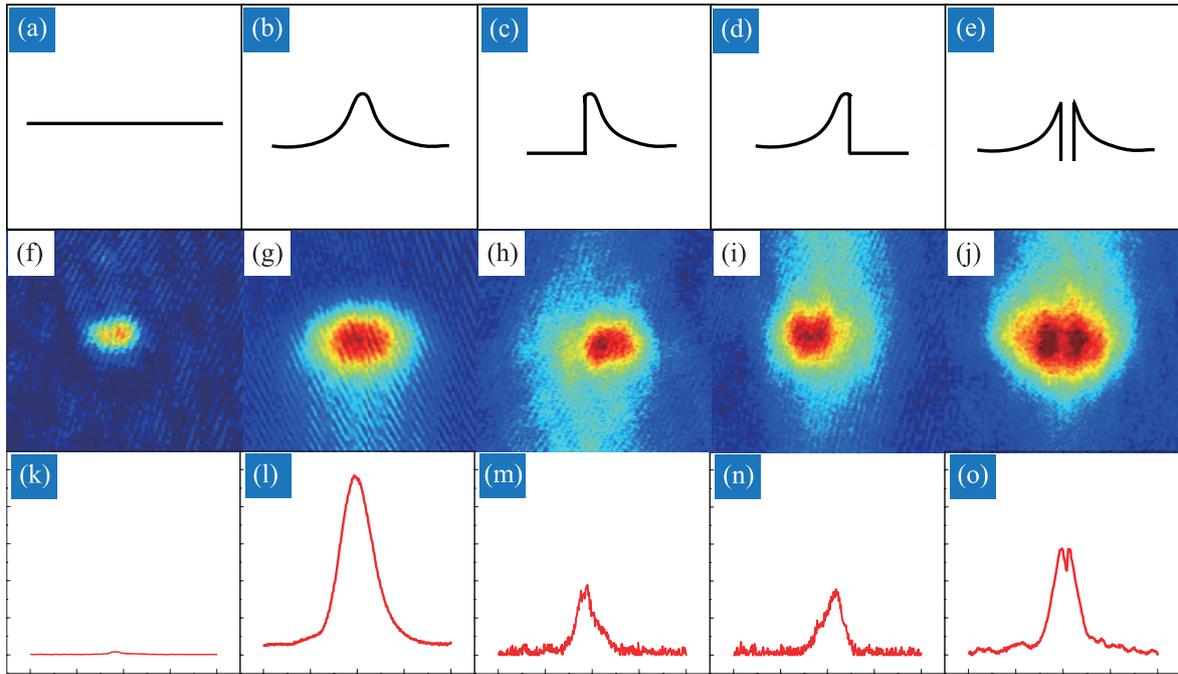


Figure 6. (a)–(e) Schematic diagram of a variety of levitation laser spot shapes, corresponding to the absorption image; (f)–(j) absorption images of the atoms taken along the horizontal direction after 2 ms of time of flight following the release of the optically levitated dipole trap; (k)–(o) the corresponding distributions of the optical intensity along the horizontal direction.

kinetic energy, $\bar{E}_{kin} = 3k_B T/2$, k_B is the Boltzmann constant, and T is the temperature of the atoms, so equation (3) can be written as

$$\dot{T} = \frac{2/3}{1 + \kappa} T_{rec} \frac{\Gamma}{\hbar |\Delta|} \left(U_0 + \frac{3\kappa}{2} k_B T \right) \quad (4)$$

where T_{rec} is the recoil temperature at the wavelength of the trapping field, and it is $0.2 \mu\text{K}$ for the Cs atoms on the D_2 line. For the red-detuned optical dipole trap, the trap depth is much larger than the thermal energy, $|U_0| \gg k_B T$, thus equation (4) can be expressed as

$$\dot{T} = \frac{2/3}{1 + \kappa} T_{rec} \frac{\Gamma}{\hbar |\Delta|} |U_0|. \quad (5)$$

From this equation, we have obtained the heating rate of the atoms at an intensity of 177 W cm^{-2} , corresponding to a detuning of -25.0 GHz . The heating by the lasers would intensify the collision of the atoms, and the collisional process can lead to substantial trap loss. The loss equation is [27]

$$\dot{N}(t) = -\alpha N(t) - \beta \int_V n^2(r, t) d^3 r \quad (6)$$

where N is the number of atoms in the dipole trap, α is the loss coefficient of the single atom collisions with the background gas, β is the two-body loss coefficient and $n(r) = n_0 \exp(-\frac{x^2}{2\sigma_x^2}) \exp(-\frac{y^2}{2\sigma_y^2}) \exp(-\frac{z^2}{2\sigma_z^2})$ is the trapped atom density distribution, with $\sigma_i = \omega_i^{-1} \sqrt{k_B T/m}$, r is the position of the atoms, and t is the time the atoms are trapped in the dipole trap. According to the simplified model in [27], the loss rate of β is $(3.67 \pm 0.6) \times 10^{-9} \text{ cm}^3 \text{ s}^{-1}$ and can be obtained

by the experimental results in figure 3. All the equations and the computing methods about heating rate and the loss rate are derived from [26].

Figure 5(a) shows the variation of the temperature of the atoms with time. The dots are experimental results and the solid line is the fitting results using the exponential function. For a conventional dipole trap, the relation between temperature and time is linear. However, the levitated laser introduces the heating of the atoms. Figure 5(b) demonstrates the heating rate as a function of time, which is simulated by equation (5). The errors come systematically from the temperature of the trapped atoms in each experimental cycle. As the time that the atoms are trapped in the optically levitated dipole trap increases, the heating rate presents a growth tendency, while the rate of growth gradually slows. Unlike the magnetically levitated dipole trap, although this limitation is inevitable with this technology, the magnetic field can be used as a degree of freedom for the manipulation of the atomic sample.

When loading cold atoms into the optically levitated dipole trap without a magnetic field, a complex spatial intensity distribution can easily be produced by a change in the spot shape of the focused levitation laser. The shapes of the levitation laser spot studied here are shown schematically in figures 6(a)–(e). Figures 6(f)–(j) show the experimental intensity distributions of the atoms trapped in (and released from) the trap for different spot shapes of the levitation laser, as visually demonstrated by the absorption images. Figures 6(k)–(o) show the corresponding distributions of the optical intensity, which have an obvious similarity to the shapes of the levitation laser spots.

5. Conclusion

In summary, the absence of the magnetic field from the loading process permits a greater number of atomic species in the trapped cloud and better control of the residual magnetic field in precision measurements, since the intensity and detuning of an optical field can be easily and rapidly changed. Furthermore, an optical field can easily be utilized to control the spatial distributions of ultracold atomic or molecular samples, and appropriate optical transitions are always available even when no magnetic levitation exists. Our method of preparing a large number of atoms in the optically levitated dipole trap opens up the possibility of efficient loading of ultracold atoms and molecules in arbitrary spin states.

In this work, we have demonstrated the efficient loading of cold atoms in a large-volume crossed optical dipole trap with the help of a levitation laser. We achieved a maximum of $\sim 5.1 \times 10^6$ Cs atoms in this optically levitated dipole trap. The optimal experimental value (to achieve the maximum number of trapped Cs atoms) for the laser intensity of 177 W cm^{-2} and detuning of -25.0 GHz agree well with the theoretically predicted values of 169 W cm^{-2} and -23.70 GHz . Compared to magnetic levitation, our optical method has several advantages. Firstly, pure state atomic samples can be obtained by this entirely optical trapping method, which does not involve the effect of a magnetic field. Secondly, the gravity of the atoms is compensated for by a focused and tunable laser beam that is more feasible than a magnetic field. This method can be adapted and implemented for the efficient loading of different species of atoms.

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