Journal of Optics

PAPER

Measurement of the wavefunction for a biphoton state with homodyne detection using least squares estimation

To cite this article: Yashuai Han *et al* 2020 *J. Opt.* **22** 025202

IOP Publishing Journal of Optics
J. Opt. **22** (2020) 025202 (7pp) https://doi.org/10.1088/2040-8986/ab6422

Measurement of the wavefunction for a biphoton state with homodyne detection using least squares estimation

Yashuai Han[1,2], Daohua Wu[3], Katsuyuki Kasai[4], Lirong Wang[5,6,7], Masayoshi Watanabe[1] and Yun Zhang[1,7]

[1] Department of Engineering Science, The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu-shi, Tokyo 182-8585, Japan
[2] Anhui Province Key Laboratory of Photo-Electronic Materials Science and Technology, College of Physics and Electronic Information, Anhui Normal University, Wuhu, Anhui, 241000, People's Republic of China
[3] School of Electrical Engineering, Anhui Polytechnic University, Wuhu, 241000, People's Republic of China
[4] Advanced ICT Research Institute, National Institute of Information and Communications Technology, 588-2, Iwaoka, Nishi-ku, Kobe, Hyogo, 651-2492, Japan
[5] State key Laboratory of Quantum optics and Quantum Optics Devices, Institute of Laser Spectroscopy, Shanxi University, Taiyuan, 030006, People's Republic of China
[6] Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan, 03006, People's Republic of China

E-mail: wlr@sxu.edu.cn and zhang@ee.uec.ac.jp

Received 9 July 2019, revised 25 November 2019
Accepted for publication 19 December 2019
Published 7 January 2020

Abstract

We report the measurement of wave function for a biphoton state with continuous variables (CV) by homodyne detection. The biphoton interference occurs on an optical parametric oscillator, which is seeded with a coherent state. A set of second-order correlation function values for the output field are recorded when the intensity coherent state is varied. The wave function, which includes both magnitude and argument for a complex parameter of the biphoton state, is reconstructed using the least square estimation method. This may introduce another approach to characterizing a biphoton state in the regime of CV.

Keywords: wave function, biphoton state, second-order correlation function, continuous variables

(Some figures may appear in colour only in the online journal)

1. Introduction

The ability to characterize quantum properties of a physical system plays a central role in the foundations of quantum physics and applications in quantum information science [1, 2]. Especially, the characterization of nonclassical properties of light such as anti-bunching, entanglement, and squeezing, is of great interest in quantum optics and constitutes a useful resource in quantum technology. In general, the information of a state can be codified in a state vector $|\psi\rangle$, or density operator $\hat{\rho}$, with which one can predict possible measurement output of observables. In order to obtain the information of the state, several approaches have been developed in both continuous variables (CV) [3] regime and discrete variables (DV) [4] regime quantum optics over past decades.

One approach is to directly measure a variance or correlation for an observable. For CV system, the measurement is usually performed by homodyne detection and the variance or correlation on quadrature amplitude of field is investigated. Meanwhile, photon statistics of light are achieved by

[7] Authors to whom any correspondence should be addressed.

measuring coincidence with single-photon detector for DV system. Although this approach is simple and intuitive, the measurement completely destroys the quantum system and some original quantum characterization could be lost. To obtain more information of the quantum system, quantum-state tomography technique [5] has been developed. This technique is based on performing a large set of distinct measurements for quadrature amplitude. Then, Wigner function in phase space and the density matrix for a state to be measured are reconstructed with mathematical algorithm. In past decades, this approach has been extensively investigated. CV quantum-state tomography has been proven an efficient method to characterize the nonclassical states, such as a single-photon Fock state [6, 7], two-photon Fock states [8], and Schrodinger's cat states [9–11]. For DV systems, the state density matrix in a discrete Hilbert space was also reconstructed through tomographic method [12–14]. Furthermore, it should be pointed out in particular that the Wigner function can be obtained with weak homodyne detection [15–17], which merges the different techniques introduced for DV and CV. Although the reconstructed Wigner function with this approach can give insight into the properties and behaviors of the state that might otherwise not be obvious, it is an indirect approach and the computations are a little complicated.

Another successful approach for characterization of a quantum state is to reconstruct the wave function by quantum interference technology [18–21] or weak measurement [22–24], providing complete knowledge of a quantum state. Owing to vast potential applications in quantum communications, quantum information processing, foundations of physics, and quantum metrology, the reconstruction of wave function of a biphoton state has attracted a great deal of attention. Using two-photon interference against an auxiliary coherent state, Beduini *et al* demonstrated a complete measurement of the complex temporal wave function of biphotons from a narrow band squeezed vacuum state [19]. Chen *et al* measured the biphoton temporal wave function generated from the spontaneous four-wave mixing in cold atoms, with polarization-dependent and time-resolved two-photon interference [20]. Tishler *et al* also reconstructed the complex spectral wave function of a biphoton produced by a type-II parametric down conversion, using quantum interference [21]. Furthermore, the position wave function of a single photon has also been obtained directly by weak measurement [22]. The above-mentioned achievements were focused on the regime of DV, using the single photon detectors as measurement instrument. The foundation for reconstruction of wave function with interferometric method lies on the measurement of second-order correlation function. Recently, it has been demonstrated that the second-order correlation function can also be measured using homodyne detection system [25, 26]. Thus, it gives possibility to reconstruct wave function in CV quantum optics regime.

In this paper, the reconstruction of wave function of a biphoton state was performed in CV regime with homodyne detection for the first time. To achieve it, we theoretically modelled and calculated the second-order correlation function for a output field of a seeded optical parametric oscillator (OPO) by introducing a loss parameter. The second-order correlation function values are measured with a pair of homodyne detectors in experiment. Both magnitude and argument of the complex parameter of the wave function for the biphoton state are reconstructed using estimation theory, which fits a set of second-order function values with least square method.

2. Model

To understand the principle, we consider a time in-dependent state $|d\rangle$ in a single mode. Its biphoton wave function can be obtained by projecting on the $|2\rangle$ state, i.e. $\phi = \langle 2|d\rangle = \langle 0|\hat{a}\hat{a}|d\rangle$. We consider the commonly encountered case that $|d\rangle$ state contains no more than two photons. Hence, the second correlation function $g^{(2)}(0) \propto \langle d|\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}|d\rangle = 2|\phi|^2$ gives important but incomplete information about the biphoton wave function, as it contains no information on the phase, which is a complex parameter. It is demonstrated that the information on the phase can be reconstructed based on the phenomenon of interference of two-photon amplitude with an auxiliary reference state. The core idea for reconstruction of wave function with interferometric method lies on the strong relationship between the measured second-order correlation function and the reference state. For example, Beduini *et al* reconstructed the biphoton wave function by measuring second-order correlation function when the phase of ancillary coherent is changed [19]. Here, we propose a model to measure the biphoton wave function basing on the change of amplitude of the coherent ancillary. The measuring state is produced by a spontaneous parametric down-conversion or an OPO. The output of OPO can be expanded in photon number representation as form of

$$|\zeta\rangle \approx |0\rangle - \frac{\zeta}{\sqrt{2}}|2\rangle, \tag{1}$$

where $\zeta = |\zeta|e^{i\theta}$ is the biphoton wave function for this state. It is a complex number and has a small magnitude. To measure it, we seed the OPO with a weak coherent state. It should be emphased that the phase θ in equation (1) means the phase of pump phase minus twice the phase of seed coherent state phase. Generally, the output state of the seeded OPO can be considered as the displaced squeezed state

$$|\psi_{out}\rangle = \hat{D}(\alpha)|\zeta\rangle = \hat{D}(\alpha)\hat{S}(\zeta)|0\rangle, \tag{2}$$

where $\hat{S}(\zeta)$ and $\hat{D}(\alpha)$ are squeezing and displacement operators, respectively. For convenience, α is chosen as a real number. The second-order correlation function for this state is given as [27]

$$g^{(2)}(0) = \frac{\langle\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}\rangle}{\langle\hat{a}^\dagger\hat{a}\rangle^2} = 1 + \frac{\cosh 2|\zeta|\sinh^2|\zeta| + |\alpha|^2[(\cosh 4|\zeta| - \cosh 2|\zeta|) - (\sinh 4|\zeta| - \sinh 2|\zeta|)\cos\theta]}{[(\cosh 2|\zeta| - \sinh 2|\zeta|\cos\theta)|\alpha|^2 + \sinh^2|\zeta|]^2}. \tag{3}$$

Differing from the previous reports, the measurement of second-order correlation function is performed in CV regime, in which a modified Hanbury–Brown–Twiss intensity interferometer with two homodyne detectors is utilized [25, 26]. In CV experiment, the values of $g^{(2)}(0)$ are calculated based on the measured quadrature variances V^+ and V^- ($V^+ = e^{2|\zeta|}$, $V^- = e^{-2|\zeta|}$). This equation is specific to a pure state ($V^+V^- = 1$). In DV quantum optics, losses do not affect the reconstruction results, and they only imply longer acquisition time in order to reach statistical significance. However, in a homodyne detection system, losses strongly affect the measurement outcomes. Usually, the loss changes a pure state into an impure state ($V^+V^- > 1$). The loss can be modelled by a beam splitter with amplitude transmission of η, which has a second input port for vacuum fluctuations. Although this simple model has some restrictions, it is enough for the present work. Hence, the quadrature variances $e^{2|\zeta|}$ and $e^{-2|\zeta|}$ are transformed to $\eta e^{2|\zeta|} + 1 - \eta$ and $\eta e^{-2|\zeta|} + 1 - \eta$, respectively. Applying the transformation to equation (3), we obtain

$$
\begin{aligned}
g^{(2)}(0) &= 1 + \frac{A_{|\zeta|,\eta} + B_{|\zeta|,\theta,\alpha,\eta} + C_{|\zeta|,\theta,\alpha,\eta}}{D^2_{|\zeta|,\theta,\alpha,\eta}},\\
A_{|\zeta|,\eta} &= 2\eta^2[(e^{2|\zeta|} - 1)^2 + (e^{-2|\zeta|} - 1)^2],\\
B_{|\zeta|,\theta,\alpha,\eta} &= 8|\alpha|^2[\eta^2(e^{2|\zeta|} - 1)^2(1 - \cos\theta)\\
&\quad + \eta^2(e^{-2|\zeta|} - 1)^2(1 + \cos\theta)],\\
C_{|\zeta|,\theta,\alpha,\eta} &= 8|\alpha|^2[\eta e^{2|\zeta|}(1 - \cos\theta)\\
&\quad + \eta e^{-2|\zeta|}(1 + \cos\theta) - 2\eta],\\
D_{|\zeta|,\theta,\alpha,\eta} &= 2|\alpha|^2[\eta e^{2|\zeta|}(1 - \cos\theta) + \eta e^{-2|\zeta|}(1 + \cos\theta)\\
&\quad + 2(1 - \eta)] + \eta(e^{2|\zeta|} + e^{-2|\zeta|} - 2).
\end{aligned} \tag{4}
$$

Here, we propose to reconstruct the magnitude $|\zeta|$ and argument θ of complex parameter ζ of the biphoton state with estimation theory, which estimates the values of parameters based on measured empirical data that has a random component. In estimation theory, there are many methods and we choose to estimate parameters with least squares. Equation (4) can be a promising model function for least squares method, in which $|\zeta|$, θ, and η are considered as free parameters. This equation enables comparison between the predicted and actually measured $g^{(2)}(0)$. In the experiment, the measured $g^{(2)}(0)$ is obtained by calculating the correlations among four permutations of measured quadrature components, which was obtained by two sets of balanced homodyne detectors. $g^{(2)}(0)$ is plotted along a range of displacements α. Based on measured $g^{(2)}(0)$ versus α, we can evaluate the values of parameters $|\zeta|$, θ, and η with least squared method. This method defines the residual as the difference between the actually measured $g^{(2)}(0)_\alpha$ value and predicted $g^{(2)}(0)^*_\alpha$ value by the theoretical model for a fixed α. Then, the sum S of squared residuals for all α is given

$$S = \sum_\alpha (g^{(2)}(0)_\alpha - g^{(2)}(0)^*_\alpha)^2. \tag{5}$$

The least square method finds the optimal parameters, minimizing the sum S of squared residuals. We set a reasonable interval for parameters $|\zeta|$, θ, and η, which is in line with physical facts. Within this interval, the optimal values of parameters $|\zeta|$, θ, and η are given by minimizing S. After this, complex parameter $\zeta = |\zeta|e^{i\theta}$ of the biphoton state is reconstructed and thus the state is characterized.

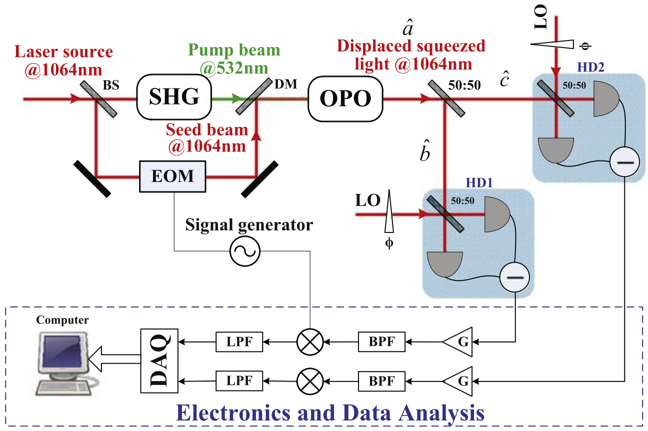

Figure 1. Schematic of experimental setup. SHG: second harmonic generation; OPO: optical parametric oscillator; EOM: electro-optics modulator; BS: beam splitter; DM: dichroic mirror; LO: local oscillator; LPF: low-pass filter; BPF: band-pass filter; $\otimes$: mixer.

3. Experimental setup

The experimental setup is outlined in figure 1. The laser source is a Nd:YAG laser, emitting 2.0 W of coherent light at 1064 nm. A major fraction of the beam was efficiently frequency-doubled in a bow-tie enhancement cavity, described in [28]. The green laser generated at 532 nm is used to pump an OPO. A small part of fundamental beam is injected into OPO, used as a seed beam. Before the injection into OPO, the weak seed beam is amplitude-modulated by an electro-optical modulator (EOM). This modulation was used to produce a coherent state on one of the analysis frequency of the laser. In our experiment, the modulation frequency is 11 MHz. The OPO cavity has a standing-wave configuration, consisting of a PPKTP crystal and two concave mirrors. For 1064 nm, the input mirror is highly reflective and the output mirror has a transmission of 10%. Both of the two mirrors are highly transmissive at 532 nm.

By varying the relative phase between pump and seed beams, the OPO cavity can be operated in the condition of parametric amplification or deamplication, in which the output state is called a phase squeezed state or amplitude squeezed state. The resulting output beam from OPO cavity is split into two beams with a 50:50 beam splitter. Each of the two beams is sent to a homodyne detection system. The electrical signals of both homodyne detectors were amplified, bandpass-filtered, mixed at 11 MHz, and low-pass filtered. Then, we use a 12-bit analog-digital-converter with a rate of 240 kS s^{-1} to sample the signals. Finally, the sampled signals were recorded on a computer for analysis.

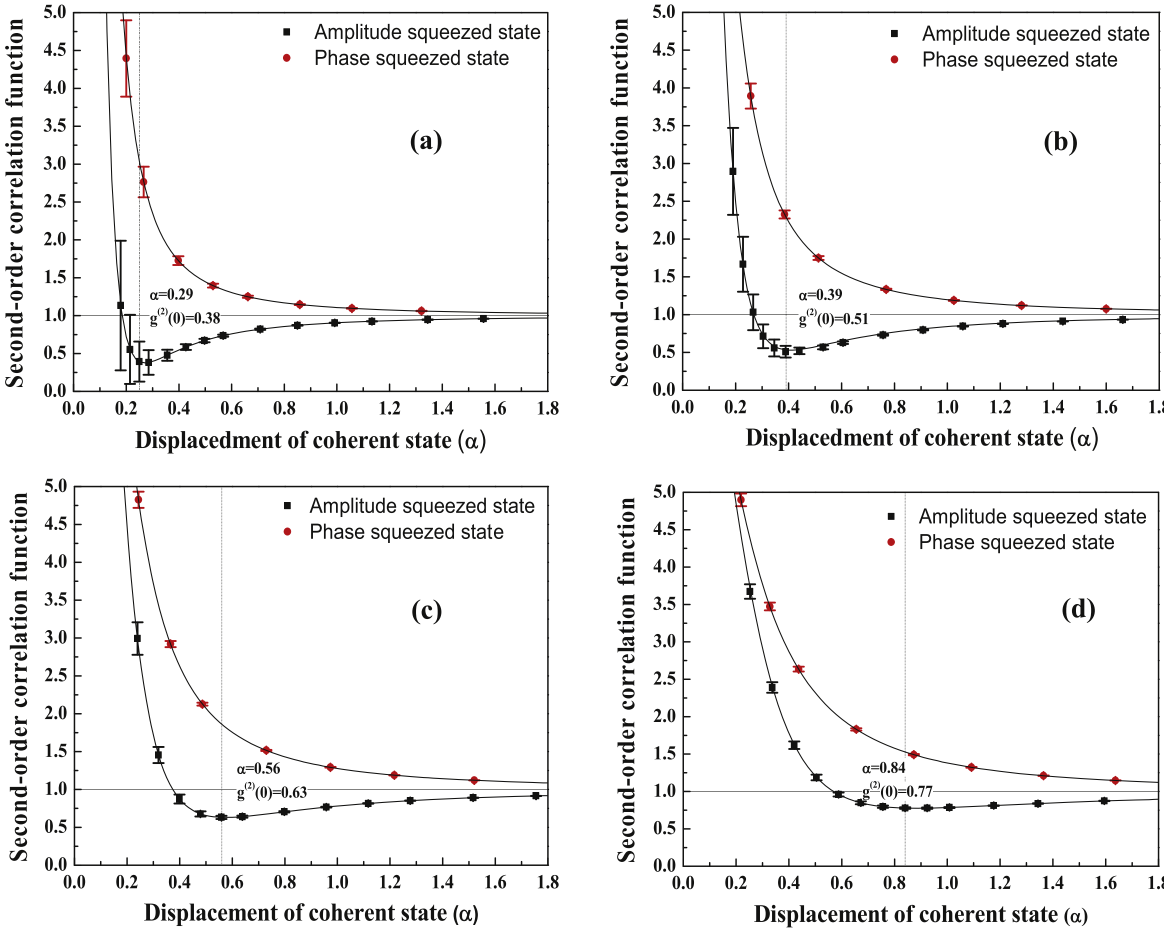

Figure 2. Experimental measurement of $g^{(2)}(0)$ as a function of displacement α of the coherent state at different pump powers $P_{2\omega}=$ (a) 14 mW, (b) 35 mW, (c) 70 mW, (d) 175 mW. The black squares are data for amplitude squeezed states, the red circles are data for phase squeezed states, and the solid lines are fitting curves with equation (5).

4. Experimental results and discussion

For calculating a second-order correlation function value, four permutations of quadratures, which include both quadrature amplitude and quadrature phase for two separated modes $\hat{b}$ and $\hat{c}$, are required. It is experimentally performed by controlling phase of local beams of the homodyne detection system. The measured mean, variance and covariance of four permutations of quadratures were used to calculate the second-order correlation function [25, 26]. Actually, the mean of amplitude represents the displacement α of a coherent state. This displacement is normalized to the variance of the vacuum state and it can be varied by adjustment of the amplitude of the modulation signal at EOM. A set of $g^{(2)}(0)$ values for both amplitude squeezed state and phase squeezed state were obtained when we verified the displacement of the coherent state or pump power for the OPO.

According to our theoretical model, a set of $g^{(2)}(0)$ values of a displaced squeezed state versus displacement α is necessary for reconstructing the wave function of the biphoton state. Figure 2 gives some examples for biphoton states with different coefficients, which are produced by operating the OPO at different pump powers of 14 mW, 35 mW, 70 mW and 175 mW, respectively. It clearly shows that the $g^{(2)}(0)$ is always greater than 1 and monotonically decreases to 1 with the increment of displacement α for phase squeezed state. This indicates super-Poissonian statistics and photon bunching effect for the phase squeezed state. On the other hand, $g^{(2)}(0)$ for the amplitude squeezed state also decreases from more than 1 to less than 1 and reaches a minimum value at a certain value of displacement α. As the displacement increases further, $g^{(2)}(0)$ monotonically grows and approaches to 1. We can find out from the four samples that the displacement α, which corresponds to the minimum $g^{(2)}(0)$, shifts to higher range for higher pump power. Its value changes from 0.29 to 0.84 when the pump rises up from 14 to 175 mW. Meanwhile, the minimum $g^{(2)}(0)$ increases from 0.38 to 0.77.

The above-discussed behaviors can be also understood from a viewpoint of two-photon interference [29–31]. The interference occurs between two-photon probability amplitudes of the biphoton state and coherent state. For a given pump power, the biphoton state has a two-photon probability amplitude of $|\zeta|$ and the coherent state has a probability amplitude of α^2. For $|\zeta| \gg \alpha^2$, the two-photon probability is dominated by the biphoton state, so both amplitude squeezed state and phase squeezed state give a $g^{(2)}(0)>1$. On the other hand, the $g^{(2)}(0)$ approximates to 1, indicating that the mixed

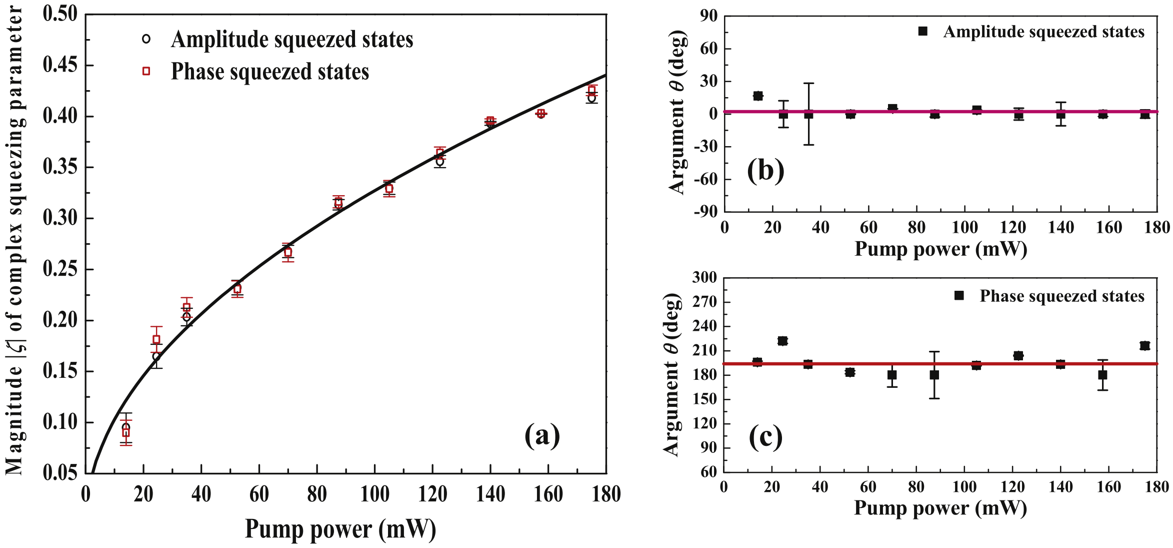

Figure 3. Reconstructed magnitude $|\zeta|$ and argument θ of the complex parameter for the biphoton state with the pump power increasing. In (a), the squares and circles are the fitted magnitude $|\zeta|$ from amplitude and phase squeezed states, curves are theoretical calculation results. (b), (c) The fitted argument θ from amplitude and phase squeezed states, respectively.

field is dominated by the coherent field for $|\zeta| \ll \alpha^2$. A clear two-photon interference is able to be observed at the range of $|\zeta| \sim \alpha^2$. Especially when $|\zeta| = \alpha^2$, we have complete destructive two-photon interference, giving a minimum value of $g^{(2)}(0)$. This can be used to interpret the displacement shift with the pump power increasing. However, from the viewpoint of two-photon interference, the minimum $g^{(2)}(0)$ should always be 0 for different pump powers, which is different from the measured results. So when only two-photon interference is considered, the increment of minimum $g^{(2)}(0)$ value versus pump powers cannot be explained. The increment of minimum $g^{(2)}(0)$ value can be explained by the following two reasons. One is its innate characterization and can be predicted with equation (4). The other one is purity of the squeezed state. In the experiment, the purity of the squeezed state usually deteriorates as the pump power rises.

The wave function of the biphoton state is reconstructed by fitting the measured $g^{(2)}(0)$ with equation (4). Parameters ζ, θ, and η are selected as free parameters and the square of difference between the measured $g^{(2)}(0)$ and predicted $g^{(2)}(0)$ is minimized by least square method. The parameters ζ and θ represent the magnitude and argument of the complex parameter for a wave function. From figure 2(a), $|\zeta| = 0.095$ and $\theta = 16.7°$ are obtained for amplitude squeezed state and $|\zeta| = 0.090$ and $\theta = 195.0°$ are achieved for phase squeezed state. We summarize the reconstructed magnitude and argument for amplitude and phase squeezed state at different pump powers in figure 3. As a matter of fact, the squeezing level and pump power has been well investigated and has the relation of [32–34]

$$|\zeta| = -\frac{1}{2}\ln\left(1 - \frac{4\sqrt{P_{2\omega}/P_{th}}}{(1 + \sqrt{P_{2\omega}/P_{th}})^2 + 4\Omega^2}\right), \tag{6}$$

in which P_{th} is the threshold of the OPO cavity, and $\Omega = 2\pi f/\gamma$ is a detuning parameter. Ω is calculated to be 0.2 using the analysis frequency f of 11 MHz and the cavity decay rate γ of $3.5 \times 10^8\ \mathrm{s}^{-1}$. P_{th} is estimated to be 2.8 W with the measured parametric gain. The curve in figure 3(a) gives predicted squeezing level using equation (6). It shows a reasonable agreement between the predicted and reconstructed squeezing parameters. Furthermore, the two reconstructed magnitudes from amplitude squeezed state and phase squeezed state also have an excellent consistency. Figures 3(b) and (c) show the reconstructed argument θ of the complex parameter versus the pump power. A constant phase offset is obtained for each case, which is expected for an ideal OPO. The average phase offsets of $2.3° \pm 5.1°$ and $194.6° \pm 14.4°$ are achieved for the amplitude squeezed state and phase squeezed state, respectively. Theoretically, the phase of amplitude and phase squeezed states should differ 180°. This deviation can be attributed to two aspects. First, as the phase squeezed states are obtained when the OPA is operated in parametric amplification, the noise of phase squeezed states is usually bigger than that of amplitude squeezed states as the noise coupling from the seed laser. The classical phase fluctuation is obviously coupled and this may cause a larger deviation for phase squeezed states. Furthermore, the locking condition for amplitude and phase states is also different. For the same seed power, the intensity of phase squeezed states is stronger than that of amplitude squeezed states. This may also induce a phase offset. A few points show the large uncertainty in the phase values, which can be attributed to accidental noise in data acquisition, for instance, the instantaneous electronic noise of the loop or vibration of the platform.

Figure 4 gives the reconstructed parameter η for amplitude and phase squeezed state. In our theoretical model, η is transmission of a fictitious beam splitter. In the experiment, η represents the total detection efficiency of the system for the biphoton state, after it is generated in the OPO cavity. This includes OPO escape efficiency, propagation efficiency, and homodyne detection efficiency, which are estimated to be 80%, 92%, 79% for our system, respectively. The total

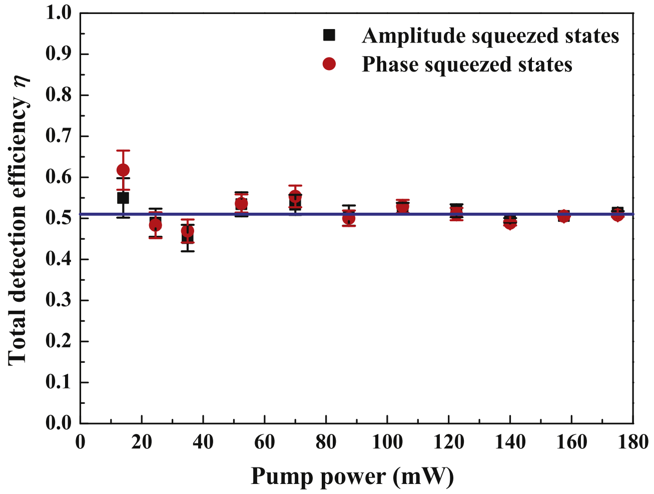

Figure 4. Reconstructed total detection efficiency η of the system versus the pump power. The black squares and red circles are the results for amplitude and phase squeezed states, respectively.

detection efficiency is calculated to be 58%. An average transmission η of 51% in figure 4 agrees well with the total detection efficiency in our experiment.

5. Conclusion

In conclusion, we theoretically proposed and experimentally demonstrated an approach for characterization of the wave function of a biphoton state via homodyne detection. This approach is based on measurement of the second-order correlation function by seeding an OPO with an auxiliary coherent state. The reconstructed wave function, which includes both magnitude and argument for a complex parameter of the biphoton state, has a reasonable agreement with the results predicted from a viewpoint of OPO. Furthermore, the detection efficiency of the system is also evaluated, which is not achieved in other related studies.

Until now, similar results were mainly achieved based on DV detection technique. This study confirms the possibility for the reconstruction of wave function of a biphoton state with CV detection technique. It may stimulate and promote the development of DV and CV quantum optics. In addition, for the further research associated with the reconstruction of three-photon, four-photon and other multi-photon states, the DV detection technique will be limited by the production rate of photons and quantum efficiency of single-photon detectors. The present technique is relatively free of this trouble and has the potential to reconstruct the wave function of multi-photon states. To summarize, we believe this approach may be useful for detecting and characterizing quantum states for quantum information processing, quantum communications, and quantum metrology.

Funding

JSPS KAKENHI (17K05071, 17K05072); National Natural Science Foundation of China (NSFC) (61875112, 61728502, 61575116); Program of State Key Laboratory of Quantum Optics and Quantum Optics Devices (KF201812); Program for Sanjin scholars of Shanxi Province; Scientific Research Foundation of Anhui Polytechnic University 2018YQQ008.

ORCID iDs

Yashuai Han https://orcid.org/0000-0002-1848-5208
Lirong Wang https://orcid.org/0000-0002-2446-0656

References

[1] O'Brien J L, Furusawa A and Vučković J 2009 Photonic quantum technologies *Nat. Photon.* **3** 687–95
[2] Lvovsky A I and Raymer M G 2002 Continuous-variable optical quantum-state tomography *Rev. Mod. Phys.* **81** 299–332
[3] Braunstein S L and van Loock P 2005 Quantum information with continuous variables *Rev. Mod. Phys.* **77** 513–77
[4] Kok P, Munro W J, Nemoto K, Ralph T C, Dowling J P and Miburn G J 2007 Linear optical quantum computing with photonic qubits *Rev. Mod. Phys.* **79** 135–74
[5] Lvovsky A I, Hansen H, Aichele T, Benson O, Mlynek J and Schiller S 2001 Quantum state reconstruction of the single-photon Fock state *Phys. Rev. Lett.* **87** 050402
[6] Zavatta A, Viciani S and Bellinil M 2004 Quantum-to-classical transition with single-photon-added coherent states of light *Science* **306** 660–2
[7] Qin Z, Prasad A S, Brannan T, MacRae A, Lezama A and Lvovsky A I 2015 Complete temporal characterization of a single photon *Light: Sci. Appl.* **4** e298
[8] Ourjoumtsev A, Tualle-Brouri R and Grangier P 2006 Quantum homodyne tomography of a two-photon Fock state *Phys. Rev. Lett.* **96** 213601
[9] Ourjoumtsev A, Jeong H, Tualle-Brouri R and Grangier P 2007 Generation of optical 'Schrödinger cats' from photon number states *Nature* **448** 784–6
[10] Wakui K, Takahashi H, Furusawa A and Sasaki M 2007 Photon subtracted squeezed states generated with periodically poled $KTiOPO_4$ *Opt. Express* **15** 3568–74
[11] Asavanant W, Nakashina K, Shiozawa Y, Yoshikawa J and Furusawa A 2017 Generation of highly pure Schrödinger's cat states and real-time quadrature measurements via optical filtering *Opt. Express* **25** 32227–42
[12] White A G, James D F V, Eberhard P H and Kwiat P G 1999 Nonmaximally entangled states: production, characterization, and utilization *Phys. Rev. Lett.* **83** 3103
[13] Adamson R B A and Steinberg A M 2010 Improving quantum state estimation with mutually unbiased bases *Phys. Rev. Lett.* **105** 030406
[14] Bernhard C, Bessire B, Feurer T and Stefanov A 2013 Shaping frequency-entangled qudits *Phys. Rev.* A **88** 032322
[15] Banaszek K, Radzewicz C and Wódkiewicz K 1999 Direct mearesurment of the Wigner function by photon counting *Phys. Rev.* A **60** 674
[16] Bondani M, Allevi A and Andreoni A 2009 Wigner function of pulsed fields by direct detection *Opt. Lett.* **34** 1444
[17] Puentes G, Lundeen J S, Branderhorst P A, Coldenstrodt-Ronge H B, Smith B J and Walmsley I A 2009 Bridging particle and wave sensitivity in a configurable detector of positive operator-valued mearsures *Phys. Rev. Lett.* **102** 080404
[18] Ren C and Hofmann H F 2011 Time-resolved measurement of the quantum states of photons using two-photon interference with short-time reference pulses *Phys. Rev.* A **84** 032108

[19] Beduini F A, Zielinska J A, Lucivero V G, de Icaza Astiz Y A and Mitchell M W 2014 Interferometric measurement of the biphoton wave function *Phys. Rev. Lett.* **113** 183602

[20] Chen P, Shu C, Guo X, Loy M M T and Du S 2015 Measuring the biphoton temporal wave function with polarization-dependent and time-resolved two-photon interference *Phys. Rev. Lett.* **114** 010401

[21] Tischler N, Büse A, Helt L G, Juan M L, Piro N, Ghosh J, Steel M J and Molina-Terriza G 2015 Measurement and shaping of biphoton spectral wave functions *Phys. Rev. Lett.* **115** 193602

[22] Lundeen J S, Sutherland B, Patel A, Stewart C and Bamber C 2011 Direct measurement of the quantum wavefunction *Nature* **474** 188

[23] Salvail J Z, Agnew M, Johnson A S, Bolduc E, Leach J and Boyd R W 2013 Full characterization of polarization states of light via direct measurement *Nat. Photon.* **7** 316

[24] Wu S 2013 State tomography via weak measurements *Sci. Rep.* **3** 1193

[25] Grosse N B, Symul T, Stobiska M, Ralph T C and Lam P K 2007 Measuring photon antibunching from continuous variable sideband squeezing *Phys. Rev. Lett.* **98** 153603

[26] Wu D, Kawamoto K, Guo X, Kasai K, Watanabe M and Zhang Y 2017 Observation of two-photon interference with continuous variables by homodyne detection *Eur. Phys. J.* D **71** 260

[27] Koashi M, Kono K, Hirano T and Matsuoka M 1993 Photon antibunching in pulsed squeezed light generated via parametric amplification *Phys. Rev. Lett.* **71** 1164–7

[28] Zhang Y, Hayashi N, Matsumori H, Mitazaki R, Xue Y, Okada-Shudo Y, Watababe M and Kasai K 2013 Generation of 1.2 W green light using a resonant cavity-enhanced second-harmonic process with a periodically poled $KTiOPO_4$ *Opt. Commun.* **294** 271–5

[29] Jin R B, Zhang J, Shimizu R, Matsuda N, Mitsumori Y, Kosaka H and Edamatsu K 2011 High-visibility nonclassical interference between intrinsically pure heralded single photons and photons from a weak coherent field *Phys. Rev.* A **83** 031805

[30] Li X, Yang L, Cui L, Ou Z Y and Yu D 2008 Observation of quantum interference between a single-photon state and a thermal state generated in optical fibers *Opt. Express* **16** 12505

[31] Li T, Sakurai S, Kasai K, Wang L, Watanabe M and Zhang Y 2018 Experimental observation of three-photon interference between a two-photon state and a weak coherent state on a beam splitter *Opt. Express* **26** 20442

[32] Bachor H A and Ralph T C 2004 *A Guide to Experiments in Quantum Optics* (Weinheim: Wiley-VCH)

[33] Mehmet M, Ast S, Eberle T, Steinlechner S, Vahlbruch H and Schnabel R 2011 Squeezed light at 1550 nm with a quantum noise reduction of 12.3 dB *Opt. Express* **19** 25763–72

[34] Takeno Y, Yukawa M, Yonezawa H and Furusawa A 2007 Observation of −9 dB quadrature squeezing with improvement of phase stability in homodyne measurement *Opt. Express* **15** 4321–7