## Letter

# Subwavelength three-dimensional Rydberg atom localization by optical absorption microscopy 

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#### Abstract

We propose a scheme for subwavelength three-dimensional (3D) Rydberg atom localization in a $(V+\Xi)$-type atomic system by spatial optical absorption microscopy. Position-dependent atom-field interaction allows atom position information to be obtained via measurement of the probe absorption. Some distinctive spatial localization patterns are discovered by adjusting the detuning and Rabi frequency of the laser fields. A $100 \%$ probability of finding the Rydberg atom at a specific position in 3D subwavelength space is achieved under appropriate conditions. This scheme may provide a novel approach for realizing high-precision 3D Rydberg atom localization in experiment.


Keywords: three-dimensional atom localization, Rydberg atom, probe absorption
(Some figures may appear in colour only in the online journal)

## 1. Introduction

In the last few decades, extensive attention has been paid to the precision measurement of atom localization, which has applications in many areas such as the trapping and cooling of atoms [1], atom nanolithography [2, 3], the coherent patterning of matter waves [4] and Bose-Einstein condensation [5]. Some methods for obtaining atomic position information have been proposed based on the spatially varying atom-field interaction and quantum interference effect [6-8], among which the use of standing-wave fields to perform accurate position measurements is an efficient way. Earlier theoretical studies mostly considered the atom localization in one dimension (1D) and two dimensions (2D). For example, 1D atom localization was studied based on the resonance fluorescence of a standingwave field [9], and by measuring the position-dependent probe
field frequency, a novel model of 1D atom localization had been researched [10]. Several high-precision 2D atom location schemes were also proposed by measuring probe absorption [11], spontaneous emission [12], population in differently configured atomic systems [13], and four-wave mixing [14].

Recently, the study of three-dimensional (3D) atom localization has started to attract research interest. Atom localization in 3D space was first discussed in five-level M-type and four-level tripod-type atomic systems based on three strong standing-wave fields, but resulted in a lower probability of finding the atom at a specific position $[15,16]$. Then a double two-level atomic system and a four-level diamond configuration atomic system were proposed to study the 3D atom localization [17, 18]. 3D atom localization in hot vapor, where the Doppler shift can seriously affect the precise spatial measurement of the atom, was theoretically studied via a


Figure 1. (a) Schematic diagram of a four-level $(V+\Xi)$-type atomic system. (b) An atom interacts with three orthogonal standing-wave fields aligning along the $x, y$ and $z$ directions to form a 3D space, while the probe $\Omega_{\mathrm{p}}$, switching $\Omega_{\mathrm{s}}$ and coupling $\Omega_{\mathrm{c}}$ fields propagate along the $z$ direction.
microwave field in a $\Lambda$-type system [19]. And there are some efficient schemes for atom localization in 3D space presented by using the probe laser field absorption [20], spontaneous emission [21] and Kerr nonlinearity [22]. However, the localization of highly excited Rydberg atoms has received little attention. The Rydberg atoms play an important role in manybody quantum physics and quantum optics information owing to their weak binding energies, large dipole moment and long radiative lifetime [23, 24]. Rydberg atoms are always generated by the two-photon excitation in a $\Xi$-type system, which allows us to study the 3D atom localization [20]. Compared with the detection methods of the Rydberg atom [25, 26], this method shows great potential in determining the accurate position of the Rydberg atom in spatial coordinates.

In this letter, we propose a scheme of 3D atom localization in a four-level $(V+\Xi)$-type atomic system using the Rydberg state as the uppermost level. The atomic position information is carried by the absorption of the probe laser field due to the position-dependent atom-field interaction. Thus, we study 3D Rydberg atom localization by measuring weak probe field spatial absorption. We discuss a robust case for achieving Rydberg atom localization in which the three orthogonal standing-wave fields are used to drive two different atomic transitions. We obtain the following main results: firstly, the isosurfaces of the probe absorption present different localization patterns when adjusting system parameters. Secondly, the maximum probability of finding the Rydberg atom is $100 \%$ in the subwavelength region. Thirdly, the measurement of probe absorption in this scheme will be much more convenient to realize experimentally. Therefore, this scheme is an effective way to realize the 3D Rydberg atom high-precision localization.

## 2. Theoretical model

In order to realize the 3D localization of Rydberg atoms, we consider a four-level $(V+\Xi)$ atomic system as illustrated in figure 1(a), where level $11>$ is the ground state, levels $12>$ and $|3\rangle$ are excited states and level $|4\rangle$ is the Rydberg state. A weak probe field with Rabi frequency $\Omega_{\mathrm{p}}$ excites atoms from level $\|>$ to level $\mid 2>$. The strong switching and control traveling-wave fields with Rabi frequencies $\Omega_{\mathrm{s}}$ and $\Omega_{\mathrm{c}}$ act on the transition of level $\mid 1>$ to level $\mid 3>$ and level $\mid 2>$ to level $14>$, respectively. Figure 1 (b) shows an atom moving along the $z$ direction interacting with three orthogonal standingwave fields in 3D space. Here, we consider the case that two standing-wave fields along the $x$ and $y$ directions couple the transition of $|1>-| 3\rangle$, and another along the $z$ direction drives the transition of $|2\rangle-14\rangle$, i.e.,

$$
\begin{align*}
& \Omega_{1}=\Omega_{x} \sin \left(k_{1} x\right)+\Omega_{y} \sin \left(k_{1} y\right)+\Omega_{s},  \tag{1}\\
& \Omega_{2}=\Omega_{z} \sin \left(k_{2} z\right)+\Omega_{c},
\end{align*}
$$

where $k_{1}$ and $k_{2}$ denote the wave vectors of the corresponding laser fields, $\Omega_{x}, \Omega_{y}$ and $\Omega_{z}$ are the Rabi frequencies of standingwave fields which propagate along the $x, y$ and $z$ directions, respectively. The $\Delta_{\mathrm{p}}, \Delta_{1}$ and $\Delta_{2}$ denote the corresponding transition frequency detunings.

The center-of-mass location of the atom along the directions of the standing-waves is assumed to be constant. We can neglect the kinetic part of the atom from the Hamiltonian in the Raman-Nath approximation, since the transverse kinetic energy of the atom is much less than the interaction energy [27]. Therefore, applying the electric dipole and rotating-wave
approximation, the interaction Hamiltonian of this system can be expressed as

$$
\begin{align*}
\mathrm{H}_{\mathrm{I}}= & \frac{\hbar}{2}\left[\Omega_{\mathrm{p}}|1><2|\right. \\
& +\Omega_{1}|1><3|+\Omega_{2}|2><4|+\hbar\left[\Delta_{p}|2><2|\right. \\
& \left.+\Delta_{1}|3><3|+\left(\Delta_{\mathrm{p}}+\Delta_{2}\right)|4><4|\right]+ \text { h.c. }, \tag{2}
\end{align*}
$$

where h.c. is the Hermitian conjugate and the equations of motion for the density-matrix elements are given by [28]

$$
\begin{align*}
\dot{\rho_{11}}= & \Gamma_{2} \rho_{22}+\Gamma_{3} \rho_{33}+\frac{i}{2} \Omega_{p}\left(\rho_{21}-\rho_{12}\right) \\
& +\frac{i}{2} \Omega_{1}\left(\rho_{31}-\rho_{13}\right), \\
\dot{\rho_{22}}= & -\Gamma_{2} \rho_{22}+\Gamma_{4} \rho_{44}+\frac{i}{2} \Omega_{p}\left(\rho_{12}-\rho_{21}\right) \\
& +\frac{i}{2} \Omega_{2}\left(\rho_{42}-\rho_{24}\right), \\
\dot{\rho_{33}}= & -\Gamma_{3} \rho_{33}+\frac{i}{2} \Omega_{1}\left(\rho_{13}-\rho_{31}\right), \\
\dot{\rho_{44}}= & -\Gamma_{4} \rho_{44}+\frac{i}{2} \Omega_{2}\left(\rho_{24}-\rho_{42}\right), \tag{3}
\end{align*}
$$

$$
\begin{aligned}
\dot{\rho_{21}} & =\gamma_{21} \rho_{21}-\frac{i}{2} \Omega_{p}\left(\rho_{22}-\rho_{11}\right)-\frac{i}{2} \Omega_{1} \rho_{23}+\frac{i}{2} \Omega_{2} \rho_{41}, \\
\dot{\rho_{31}} & =\gamma_{31} \rho_{31}-\frac{i}{2} \Omega_{1}\left(\rho_{33}-\rho_{11}\right)-\frac{i}{2} \Omega_{p} \rho_{32} \\
\dot{\rho_{41}} & =\gamma_{41} \rho_{41}+\frac{i}{2} \Omega_{2} \rho_{21}-\frac{i}{2} \Omega_{p} \rho_{42}-\frac{i}{2} \Omega_{1} \rho_{43} \\
\dot{\rho_{32}} & =\gamma_{32} \rho_{32}+\frac{i}{2} \Omega_{1} \rho_{12}-\frac{i}{2} \Omega_{p} \rho_{31}-\frac{i}{2} \Omega_{2} \rho_{34} \\
\dot{\rho_{42}} & =\gamma_{42} \rho_{42}-\frac{i}{2} \Omega_{2}\left(\rho_{44}-\rho_{22}\right)-\frac{i}{2} \Omega_{p} \rho_{41} \\
\dot{\rho_{43}} & =\gamma_{43} \rho_{43}+\frac{i}{2} \Omega_{2} \rho_{23}-\frac{i}{2} \Omega_{1} \rho_{41}
\end{aligned}
$$

with $\quad \gamma_{21}=-\frac{\Gamma_{2}}{2}+i \Delta_{\mathrm{p}}, \quad \gamma_{31}=-\frac{\Gamma_{3}}{2}+i \Delta_{1}, \quad \gamma_{41}=-\frac{\Gamma_{4}}{2}+$ $i\left(\Delta_{\mathrm{p}}+\Delta_{2}\right), \quad \gamma_{32}=-\frac{\Gamma_{3}+\Gamma_{2}}{2}-i\left(\Delta_{1}-\Delta_{\mathrm{p}}\right), \gamma_{42}=-\frac{\Gamma_{4}+\Gamma_{2}}{2}+$ $i \Delta_{2}, \gamma_{43}=-\frac{\Gamma_{4}+\Gamma_{3}}{2}+i\left(\Delta_{2}+\Delta_{\mathrm{p}}-\Delta_{1}\right)$, where $\Gamma_{i}(i=2,3,4)$ is the decay rate of the state $\mid i>$. The above density matrix elements obey the normalization and Hermitian condition $\sum_{i=1}^{4} \rho_{i i}=1$ and $\rho_{i j}^{*}=\rho_{j i}$. Assuming that the atom is initially in its ground level, under the weak probe field approximation, the steady-state solution for $\rho_{21}$ is given by

$$
\begin{equation*}
\rho_{21}=\frac{i \Omega_{p}\left(\gamma_{32} \Omega_{1}^{2}+\gamma_{41} \Omega_{2}^{2}+4 \gamma_{32} \gamma_{41} \gamma_{43}\right)}{\Omega_{1}^{2} \Omega_{2}^{2}-\Omega_{1}^{4}-\Omega_{2}^{4}-2 \Omega_{1}^{2}\left(\gamma_{21} \gamma_{32}+\gamma_{41} \gamma_{43}\right)-3 \Omega_{2}^{2}\left(\gamma_{21} \gamma_{41}+\gamma_{32} \gamma_{43}\right)-8 \gamma_{21} \gamma_{32} \gamma_{41} \gamma_{43}} . \tag{4}
\end{equation*}
$$

The susceptibility of this weak probe field is related to the term $\rho_{21}$ and can be written as

$$
\begin{equation*}
\chi=\frac{\mathrm{N}\left|\mu_{21}\right|^{2}}{\varepsilon_{0} \hbar \Omega_{p}} \rho_{21} \tag{5}
\end{equation*}
$$

where $N$ is the atomic density, $\mu_{21}$ is the transition dipole momentum between levels $\mid 2>$ and $\mid 1>$ and $\varepsilon_{0}$ denotes the permittivity of free space. The imaginary part of susceptibility, which is directly related to the probe laser field absorption, reflects the position probability distribution of the atom when it passes through the standing-wave fields in 3D subwavelength space. Therefore, the position information of the atom in 3D space can be directly obtained by measuring the filter function

$$
\begin{equation*}
F(x, y, z)=\operatorname{Im}(\chi) \tag{6}
\end{equation*}
$$

## 3. Results and discussion

In this section, we investigate the localization conditions of the Rydberg atom by numerical calculation according to equation (6). We study the case that two orthogonal standing-wave fields couple the atomic transition
$|1>-| 3\rangle$, and another couples the transition $|2>-14\rangle$, i.e. $\Omega_{1}=\Omega_{x} \sin \left(k_{1} x\right)+\Omega_{y} \sin \left(k_{1} y\right)+\Omega_{s}$ and $\Omega_{2}=\Omega_{z} \sin \left(k_{2} z\right)+\Omega_{c}$. The isosurfaces of filter function $F(x, y, z)=0.1$ versus positions ( $k_{1} x, k_{1} y, k_{2} z$ ) for different combinations of three standing-wave fields Rabi frequencies and probe laser detuning ( $\Omega_{x}, \Omega_{y}, \Omega_{z}, \Delta_{p}$ ) are shown in figure 2 , where $\Omega_{s}=\Omega_{c}=0$. As we can see from figures 2(a) and (c), when $\left(\Omega_{x}, \Omega_{y}, \Omega_{z}, \Delta_{p}\right)=\left(3 \Gamma_{2}, 3 \Gamma_{2}, 5 \Gamma_{2}, 1 \Gamma_{2}\right)$, the filter function displays eight bowl-like patterns situated at the subspaces $\left(-1 \leq k_{1} x / \pi \leq 0,-1 \leq k_{1} y / \pi \leq 0,-1 \leq k_{2} z / \pi \leq 1\right) \quad$ and $\left(0 \leq k_{1} x / \pi \leq 1,0 \leq k_{1} y / \pi \leq 1,-1 \leq k_{2} z / \pi \leq 1\right)$. The patterns of the filter function evolve to four ellipsoids when the probe field detuning increases to $\Delta_{p}=5 \Gamma_{2}$ and the Rabi frequency of the three standing-wave fields remains unchanged. In figure $2(\mathrm{~b}),\left(\Omega_{x}, \Omega_{y}, \Omega_{z}, \Delta_{p}\right)=\left(5 \Gamma_{2}, 5 \Gamma_{2}, 2 \Gamma_{2}, 3 \Gamma_{2}\right)$, the patterns are located in the same subspaces with four circular rings. However, as shown in figure 2(d), the two circular rings in each subspace become a lantern-like pattern when $\Delta_{p}=7 \Gamma_{2}$. The probe absorption of the medium will be altered by varying the probe detuning, since the $\Delta_{p}$ has a strong correlation with the filter function according to equations (5) and (6). Then, we find that the two lantern-like patterns in figure 2(e) become wider in the two subspaces when only increasing the standing-wave Rabi frequency $\Omega_{z}$ based on figure 2(d). The Rabi frequencies of the standing-wave fields also affect the weak probe field absorption by altering the


Figure 2. Isosurface of filter function $F(x, y, z)=0.1$ versus $\left(k_{1} x, k_{1} y, k_{2} z\right)$ for different combinations of three standing-wave fields Rabi frequencies and probe laser detuning $\left(\Omega_{x}, \Omega_{y}, \Omega_{z}, \Delta_{p}\right)$. (a) $\left(\Omega_{x}, \Omega_{y}, \Omega_{z}, \Delta_{p}\right)=\left(3 \Gamma_{2}, 3 \Gamma_{2}, 5 \Gamma_{2}, 1 \Gamma_{2}\right)$; (b) $\left(\Omega_{x}, \Omega_{y}, \Omega_{z}, \Delta_{p}\right)=\left(5 \Gamma_{2}, 5 \Gamma_{2}\right.$, $\left.2 \Gamma_{2}, 3 \Gamma_{2}\right)$; (c) $\left(\Omega_{x}, \Omega_{y}, \Omega_{z}, \Delta_{p}\right)=\left(3 \Gamma_{2}, 3 \Gamma_{2}, 5 \Gamma_{2}, 5 \Gamma_{2}\right)$; (d) $\left(\Omega_{x}, \Omega_{y}, \Omega_{z}, \Delta_{p}\right)=\left(5 \Gamma_{2}, 5 \Gamma_{2}, 2 \Gamma_{2}, 7 \Gamma_{2}\right)$; (e) $\left(\Omega_{x}, \Omega_{y}, \Omega_{z}, \Delta_{p}\right)=\left(5 \Gamma_{2}, 5 \Gamma_{2}\right.$, $5 \Gamma_{2}, 7 \Gamma_{2}$. Here, $\Delta_{1}=\Delta_{2}=10 \Gamma_{2}, \Omega_{s}=\Omega_{c}=0, \Gamma_{3}=0.97 \Gamma_{2}$ and $\Gamma_{4}=0.05 \Gamma_{2}$.
spatial interference of atoms with standing-wave fields in different subspaces. Compared to the simulation results in figure 2 , we decide to carry out the next work according to the parameters in figure 2(d), since its variation is more obvious under the change of other parameters.

Next, we studied the dependence of two traveling-wave fields Rabi frequencies on the 3D Rydberg atom localization. Figure 3 shows the isosurfaces of filter function $F(x, y, z)=$ 0.1 versus positions ( $k_{1} x, k_{1} y, k_{2} z$ ) with different travelingwave field Rabi frequencies $\Omega_{\mathrm{s}}$ and $\Omega_{\mathrm{c}}$. In figure 3(a), the Rabi frequencies of switching and control fields are $\Omega_{\mathrm{s}}=1 \Gamma_{2}$ and $\Omega_{\mathrm{c}}=0$, the lantern-like pattern becomes wider in the subspace $\left(0 \leq k_{1} x / \pi \leq 1,0 \leq k_{1} y / \pi \leq 1,-1 \leq k_{2} z / \pi \leq 1\right)$, while the pattern in the other subspace becomes noticeably
narrower. Interestingly, the lantern-like patterns in the two subspaces become wider in $0 \leq k_{2} z / \pi \leq 1$ while becoming narrower in $-1 \leq k_{2} z / \pi \leq 0$ when $\Omega_{s}=0$ and $\Omega_{c}=2 \Gamma_{2}$. These results can be seen from figure 3(b). This is because the localization patterns will change in the $x$ and $y$ directions by applying an increasing switching field due to $\Omega_{1}=\Omega_{x} \sin \left(k_{1} x\right)+\Omega_{y} \sin \left(k_{1} y\right)+\Omega_{s}$. Similarly, since $\Omega_{2}=\Omega_{z} \sin \left(k_{2} z\right)+\Omega_{c}$, the localization patterns will change in the $z$ direction when increasing the control field Rabi frequency. Further, the pattern in the subspace ( $0 \leq k_{1} x / \pi \leq 1,0 \leq k_{1} y / \pi \leq 1,-1 \leq k_{2} z / \pi \leq 1$ ) becomes wider while in the other subspace it becomes narrower, meanwhile the patterns become wider and narrower in $0 \leq k_{2} z / \pi \leq 1$ and $-1 \leq \mathrm{k}_{2} \mathrm{z} / \pi \leq 0$, respectively, which


Figure 3. Isosurface of filter function $F(x, y, z)=0.1$ versus $\left(k_{1} x, k_{1} y, k_{2} z\right)$ for different two traveling-wave field Rabi frequencies $\Omega_{\mathrm{s}}$ and $\Omega_{\mathrm{c}}$. (a) $\left(\Omega_{s}, \Omega_{c}\right)=\left(1 \Gamma_{2}, 0\right)$; (b) $\left(\Omega_{s}, \Omega_{c}\right)=\left(0,2 \Gamma_{2}\right)$; (c) $\left(\Omega_{s}, \Omega_{c}\right)=\left(1 \Gamma_{2}, 2 \Gamma_{2}\right)$; (d) $\left(\Omega_{s}, \Omega_{c}\right)=\left(2 \Gamma_{2}, 1 \Gamma_{2}\right)$. The other parameters are the same as figure 2(d).


Figure 4. Isosurface of filter function $F(x, y, z)=0.1$ versus $\left(k_{1} x, k_{1} y, k_{2} z\right)$ for different combinations of $\Delta_{1}$ and $\Delta_{2}$.
(a) $\left(\Delta_{1}, \Delta_{2}\right)=\left(10 \Gamma_{2}, 12 \Gamma_{2}\right)$; (b) $\left(\Delta_{1}, \Delta_{2}\right)=\left(11 \Gamma_{2}, 9 \Gamma_{2}\right)$; (c) $\left(\Delta_{1}, \Delta_{2}\right)=\left(11 \Gamma_{2}, 10 \Gamma_{2}\right)$; (d) $\left(\Delta_{1}, \Delta_{2}\right)=\left(11 \Gamma_{2}, 11 \Gamma_{2}\right)$;
(e) $\left(\Delta_{1}, \Delta_{2}\right)=\left(12 \Gamma_{2}, 12 \Gamma_{2}\right)$; (f) $\left(\Delta_{1}, \Delta_{2}\right)=\left(13 \Gamma_{2}, 13 \Gamma_{2}\right)$. The other parameters are the same as figure 3(d).


Figure 5. (a)-(c) The views of $x-y, y-z$, and $z-x$ of figure 4(f), respectively.
can be seen in figure 3(c) with $\Omega_{s}=1 \Gamma_{2}$ and $\Omega_{c}=2 \Gamma_{2}$. When the two traveling-wave field Rabi frequencies simultaneously detune to $\Omega_{s}=2 \Gamma_{2}$ and $\Omega_{c}=1 \Gamma_{2}$, the localization pattern in the subspace $\left(-1 \leq k_{1} x / \pi \leq 0,-1 \leq\right.$ $k_{1} y / \pi \leq 0,-1 \leq k_{2} z / \pi \leq 1$ ) shrinks to an ellipsoid as shown in figure 3(d). In this situation, the uncertainty of conditional position probability decreases and the precise localization of the Rydberg atom will be easier to realize.

We also investigate the influence of $\Delta_{1}$ and $\Delta_{2}$ on the 3D Rydberg atom localization behaviors based on the result in figure 3(d). In figure 4, we plot the isosurfaces of filter function $F(x, y, z)=0.1$ versus positions $\left(k_{1} x, k_{1} y, k_{2} z\right)$ for different combinations of $\Delta_{1}$ and $\Delta_{2}$. As seen from figure 4(a), the isosurface of the filter function shows almost the same patterns as in figure $3(\mathrm{~d})$ when $\Delta_{1}=10 \Gamma_{2}$ and $\Delta_{2}=12 \Gamma_{2}$. However, there is only one ellipsoid-like pattern with a concave shape located at the subspace $\left(0 \leq k_{1} x / \pi \leq\right.$ $1,0 \leq k_{1} y / \pi \leq 1,0 \leq k_{2} z / \pi \leq 1$ ) when we increase $\Delta_{1}$ to $11 \Gamma_{2}$ and decrease $\Delta_{2}$ to $9 \Gamma_{2}$ in figure $4(\mathrm{~b})$, which means the Rydberg atom can be found in such a position with a $100 \%$ probability. The concave surface will become shallower with the increasing $\Delta_{2}$, and we find a pattern similar to figure 3 (d) in the subspace ( $0 \leq k_{1} x / \pi \leq 1,0 \leq k_{1} y / \pi \leq$ $1,-1 \leq k_{2} z / \pi \leq 1$ ), while there is no pattern in any other subspace when $\Delta_{2}=11 \Gamma_{2}$, which can be seen from figure 4(d). More surprisingly, there is only an ellipsoid situated at subspace ( $0 \leq k_{1} x / \pi \leq 1,0 \leq k_{1} y / \pi \leq 1,0 \leq k_{2} z / \pi \leq 1$ ) for the case of $\left(\Delta_{1}, \Delta_{2}\right)=\left(12 \Gamma_{2}, 12 \Gamma_{2}\right)$ in figure 4(e). The ellipsoid in the subspace obviously becomes smaller when $\left(\Delta_{1}, \Delta_{2}\right)=$ $\left(13 \Gamma_{2}, 13 \Gamma_{2}\right)$, as shown in figure 4(f), which indicates that the 3D Rydberg atom localization precision is significantly improved.

To further demonstrate the precision in the determination of the atom position by this scheme, we give the dimension of the ellipsoid of figure 4(f), as shown in figure 5. Figures 5(a)-(c) are the views of $x-y, y-z$, and $z-$ $x$ of figure 4(f), respectively. The spatial resolution of the 3D Rydberg atom in this scheme is about $0.039 \lambda_{1} \times 0.039 \lambda_{1} \times 0.038 \lambda_{2}$ under the condition of filter function $F(x, y, z)=0.1$, which has a high precision for realizing the 3D Rydberg atom localization.

Before concluding, an experimental realization of this fourlevel $(V+\Xi)$-type Rydberg atomic system was proposed in cold ${ }^{85} \mathrm{Rb}$ atoms. The suggested states are as follows:
$\left|1>=\left|5 S_{1 / 2}, F=3>,\left|2>=\left|5 P_{3 / 2}, F=4>,|3>=| 5 P_{1 / 2}, F=3>\right.\right.\right.\right.$ and $|4\rangle$ is the $\mid 44 D_{5 / 2}>$ Rydberg state. The decay rates of these states are $\Gamma_{1}=0, \Gamma_{2}=2 \pi \times 6.1 \mathrm{MHz}, \Gamma_{3}=2 \pi \times 5.9 \mathrm{MHz}$ and $\Gamma_{4}=2 \pi \times 0.3 \mathrm{MHz}$, respectively $[29,30]$. To eliminate the Doppler broadening effect, the ${ }^{85} \mathrm{Rb}$ atoms in practical experiments should be trapped and cooled in a 3D magneto-optical trap (MOT) with a density of $N \sim 10^{10} \mathrm{~cm}^{-3}$ and temperature of $T \sim 100 \mu \mathrm{~K}$. The transition of $11>-12>$ can be coupled by a weak probe field with wavelength 780 nm , while the switching field and two orthogonal standing-wave fields with the same wavelength of 795 nm can couple the transition of $|1\rangle-|3\rangle$. And a control field and a standing-wave field at the wavelength of 480 nm can drive the atoms from state $13>$ to a Rydberg state $14>$. The external cavity diode lasers can provide these laser fields that are required for the experiment.

## 4. Conclusion

In conclusion, we have discussed 3D Rydberg atom localization by measuring the weak probe field spatial absorption in a four-level $(V+\Xi)$-type atomic system. The numerical simulation results indicate that the 3D Rydberg atom localization is sensitive to the Rabi frequency and detuning of the laser fields. When three orthogonal standing-wave fields are coupled to two different atomic transitions, the isosurfaces of the filter function present various location patterns, such as bowl-like, ellipsoid, circular ring and lantern-like etc, by adjusting system parameters. We realized a $100 \%$ determination probability of the Rydberg atom in a specific position with subwavelength precision. This work provides a potential method to detect the precise position of the Rydberg atom in experiment, and further promotes the applications of the Rydberg atom in the areas of quantum information and many-body physics.

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