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Application of non-Hermitian Hamiltonian model in open quantum optical systems*

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Non-Hermitian systems have observed numerous novel phenomena and might lead to various applications. Unlike standard quantum physics, the conservation of energy guaranteed by the closed system is broken in the non-Hermitian system, and the energy can be exchanged between the system and the environment. Here we present a scheme for simulating the dissipative phase transition with an open quantum optical system. The competition between the coherent interaction and dissipation leads to the second-order phase transition. Furthermore, the quantum correlation in terms of squeezing is studied around the critical point. Our work may provide a new route to explore the non-Hermitian quantum physics with feasible techniques in experiments.

Keywords: non-Hermitian Hamiltonian, open quantum system, optical parametric processing

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1. Introduction

In standard quantum mechanics, a closed system is described by the Hermitian Hamiltonian and the associated real eigenvalues spectrum. However, there always exist channels between the system and the environment, which are built through the force driving, dissipation, or fluctuation induced-dissipation. Such a system is generally called an open quantum system.[1–3] Open systems with dissipation have been a subject of significant interest in physics and can be described by non-Hermitian Hamiltonians in contrast to standard quantum mechanics requiring Hermiticity. Exceptional points (EP), known as non-Hermitian degeneracies or branch points, correspond to special points in parameter space at which both the eigenvalues and the eigenvectors of the non-Hermitian Hamiltonian simultaneously coalesce. A number of intriguing physical effects related to EP have been observed in optical gain-loss systems, such as topological chirality, potentially enhanced sensitivity, and phase transition in the case of PT-symmetry breaking.[4–8] When describing the open system of interaction between a light field and a multi-level system, the decay term is often introduced into the Hamiltonian to reflects spontaneous emission or ionization and the decay of the system population.[3,9–11] In the optical parametric processing, the dissipation caused by the interaction between the environment and the cavity reflects the lifetime of the corresponding eigenstate.[12] Quantum decoherence is also the result of entanglement between the open quantum system and environment.[13–16]

With the study of open systems, quantum mechanics has developed from Hermitian to non-Hermitian, as shown in Fig. 1. Quantum mechanics, born in the early 20th century, requires observable measurements to have real eigenvalues. The Hermitian is a sufficient and unnecessary condition for the system to have real eigenvalues. According to the PT symmetry theory defined by Bender in 1998, the observables of non-Hermitian systems with real eigenvalues need to satisfy the following three conditions in the case of even inversion symmetry:[17–19] 1. The Hamiltonian is PT symmetric (i.e., $\left[\hat{H},\hat{P}\hat{T}\right]=0$). 2. The PT symmetry of the system is not broken (i.e., $\hat{P}\hat{T}\varphi_i=\varphi_i$). 3. The Hamiltonian has the property of transposition invariance (i.e., $\hat{H}=\widetilde{\hat{H}}$). In 2002, Mostafazadeh thought that the PT symmetry theory did not include all non-Hermitian systems with real energy spectrum, which was not a necessary and sufficient condition for non-Hermitian systems to have real eigenvalues.[20] Moreover, PT-symmetric non-Hermitian Hamiltonian with transposition invariance is only a special case of pseudo-Hermitian Hamiltonian. The pseudo-Hermitian system he redefined is a non-Hermitian Hamiltonian system with a discrete energy spectrum and a set of biorthogonal basis vectors. The Hamiltonian satisfies the equation $\hat{H}^{\dagger}=\hat{\eta}\hat{H}\hat{\eta}^{-1}$, the energy spectrum of which appears in the form of real number or conjugate complex number pairs. Moreover, a necessary and sufficient condition for a pseudo-Hermitian system to have a real en-

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ergy spectrum is that there exists a linear invertible operator $\hat{O}$ such that the Hamiltonian is $\hat{O}\hat{O}^{\dagger}$ pseudo-Hermitian symmetric, i.e., $\hat{H}^{\dagger}=(\hat{O}\hat{O}^{\dagger})\hat{H}(\hat{O}\hat{O}^{\dagger})^{-1}$. In general, the dynamic evolution of the open quantum system is described by the master equation for density matrix of the system, while to study the generation of the squeezed light by optical parametric oscillator or quantum interface between light and atoms, quantum Langevin equations of motion for operators are adopted in this paper.[21–25]

In this article, we apply the non-Hermitian quantum theory in the open optical quantum system, where the dissipation is included in the Hamiltonian. Dissipative phase transition is observed and is attributed to the competition between the coherent interaction and dissipation. Furthermore, quantum correlation in terms of squeezing is analyzed here.

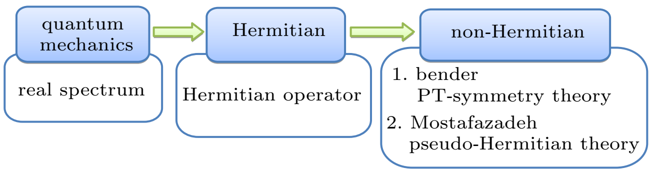

Fig. 1. The schematic of the quantum mechanics developing from Hermitian to non-Hermitian.

2. Theoretical simulation of an open quantum system

Figure 2 illustrates the schematics of the optical system considered here, in which the optical parametric interaction occurs inside an optical resonator via a nonlinear crystal.[26,27] External coherent pump light with the power of ε_{p} and seed light is coupled to the cavity through the input mirror. There are three resonant modes in the cavity: pump mode a_0 with a frequency of ω_0, and seed modes consisting of signal mode $\hat{a}_1$ and idle mode $\hat{a}_2$ with frequencies of ω_1 and ω_2, respectively. The transmission loss rates of the three modes in the cavity are assumed to be β_0, β_1, and β_2, respectively. Given the nearly degenerated signal and idle modes in the optical resonator, one can assume the equal loss for them, i.e., $\beta_1=\beta_2=\beta$. The real number χ is the nonlinear factor, proportional to the second-order polarization rate of the crystal.

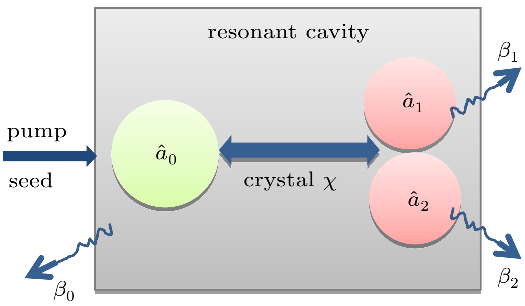

Fig. 2. Schematic of the open quantum optical system considered. One pump photon $\hat{a}_0$ converts into a pair of lower-energy photons, $\hat{a}_1$ and $\hat{a}_2$ subject to both energy conservation and the cavity resonance condition. The corresponding transmission loss rates are $\beta_i\,(i=0,1,2)$. χ is the nonlinear factor.

The non-Hermitian Hamiltonian of such an open system can be expressed as follows:

$$\begin{aligned}\hat{H} =& \sum_{i=0,1,2}\hbar\omega_i\hat{a}_i^{\dagger}\hat{a}_i+\mathrm{i}\hbar\chi\left(\hat{a}_0\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}-\hat{a}_0^{\dagger}\hat{a}_1\hat{a}_2\right)\\ &+\mathrm{i}\hbar\left(\varepsilon_{\mathrm{p}}\hat{a}_0^{\dagger}-\varepsilon_{\mathrm{p}}^{*}\hat{a}_0\right)-\sum_{i=0,1,2}\mathrm{i}\hbar\beta_i\hat{a}_i^{\dagger}\hat{a}_i, \end{aligned}\tag{1}$$

where the first term is Hamiltonian of all the optical modes, and the second term represents the optical parametric interaction term of three resonant modes in the cavity. The third term denotes the transformation between pump field ε_{p} and cavity pump mode $\hat{a}_0$, and most importantly, the fourth term represents the transmission loss of three cavity modes, i.e., non-Hermitian terms. Since we consider the steady state of the light field ($\omega_i=0,\ i=0,1,2$), the mean field approximation, $\hat{a}_i=a_i(t)\,\mathrm{e}^{-\mathrm{i}\omega_i t}=[a_i+\delta a_i(t)]\,\mathrm{e}^{-\mathrm{i}\theta_i}\,\mathrm{e}^{-\mathrm{i}\omega_i t}$ $(i=0,1,2)$ and $\varepsilon_{\mathrm{p}}=|\varepsilon_{\mathrm{p}}|\,\mathrm{e}^{-\mathrm{i}\omega_0 t}\,\mathrm{e}^{-\mathrm{i}\theta_0}$ can be used to linearize $\hat{H}$ with the mean values of $\hat{a}_i$ obtained by the steady state approximation ($\mathrm{d}\hat{a}_i/\mathrm{d}t=0$).[26,27] The quantum fluctuations higher than the second-order are ignored as small quantities. Hence, the second-order quantum fluctuation term is obtained ($\hbar=1$)

$$\delta\hat{H}=-\sum_{i=1,2}\mathrm{i}\beta_i\delta\hat{a}_i^{\dagger}\delta\hat{a}_i+\mathrm{i}\chi\alpha_0\left(\delta\hat{a}_1^{\dagger}\delta\hat{a}_2^{\dagger}-\delta\hat{a}_1\delta\hat{a}_2\right).\tag{2}$$

In order to decouple the part related to $\delta\hat{a}_1$ from $\delta\hat{a}_2$, we use the transformation $\delta\hat{d}_1=(\delta\hat{a}_1-\delta\hat{a}_2)/\sqrt{2}$ and $\delta\hat{d}_2=(\delta\hat{a}_1+\delta\hat{a}_2)/\sqrt{2}$. Then Eq. (2) reads

$$\begin{aligned}\delta\hat{H} =& -\mathrm{i}\beta\delta\hat{d}_1^{\dagger}\delta\hat{d}_1+\frac{\mathrm{i}\chi\alpha_0}{2}\left(\delta\hat{d}_1^{2}-\delta\hat{d}_1^{\dagger 2}\right)-\mathrm{i}\beta\delta\hat{d}_2^{\dagger}\delta\hat{d}_2\\ &-\frac{\mathrm{i}\chi\alpha_0}{2}\left(\delta\hat{d}_2^{2}-\delta\hat{d}_2^{\dagger 2}\right).\end{aligned}\tag{3}$$

Now we denote the terms related to $\delta\hat{d}_1$ and $\delta\hat{d}_2$ as $\delta\hat{H}_1$ and $\delta\hat{H}_2$ respectively as

$$\begin{aligned}&\delta\hat{H}=\delta\hat{H}_1+\delta\hat{H}_2,\\ &\delta\hat{H}_1=-\mathrm{i}\beta\delta\hat{d}_1^{\dagger}\delta\hat{d}_1+\frac{\mathrm{i}\chi\alpha_0}{2}\left(\delta\hat{d}_1^{2}-\delta\hat{d}_1^{\dagger 2}\right),\\ &\delta\hat{H}_2=-\mathrm{i}\beta\delta\hat{d}_2^{\dagger}\delta\hat{d}_2-\frac{\mathrm{i}\chi\alpha_0}{2}\left(\delta\hat{d}_2^{2}-\delta\hat{d}_2^{\dagger 2}\right).\end{aligned}\tag{4}$$

Regarding $\delta\hat{H}_1$, we define $n=\mathrm{i}\beta$, $m=\mathrm{i}\chi\alpha_0$, and then

$$\delta\hat{H}_1=-n\delta\hat{d}_1^{\dagger}\delta\hat{d}_1+\frac{m}{2}\left(\delta\hat{d}_1^{2}-\delta\hat{d}_1^{\dagger 2}\right).\tag{5}$$

In the matrix form of $\delta\hat{H}_1$, $\delta\hat{d}_1^{\dagger}\delta\hat{d}_1$ corresponds to diagonal elements, while $\delta\hat{d}_1^{2}$ and $\delta\hat{d}_1^{\dagger 2}$ correspond to non-diagonal elements. In order to diagonalize $\delta\hat{H}_1$, we employ the complex Bogoliubov transformations, $\delta\hat{d}_1=\hat{b}\cosh\frac{\theta}{2}+\mathrm{i}\bar{b}\sinh\frac{\theta}{2}$, $\delta\hat{d}_1^{\dagger}=\bar{b}\cosh\frac{\theta}{2}-\mathrm{i}\hat{b}\sinh\frac{\theta}{2}$. $\delta\hat{H}_1$ is transformed into[28–32]

$$\begin{aligned}\delta H_1 =& \left[(\bar{b}^2-\hat{b}^{\dagger 2})\left(-\frac{n}{2}\mathrm{i}\sinh\theta-\frac{m}{2}\cosh\theta\right)\right]\\ &+\bar{b}\hat{b}\,(-n\cosh\theta+m\mathrm{i}\sinh\theta)\end{aligned}$$

$$+\left[\frac{1}{2}(-n\cosh\theta+m\,\mathrm{i}\sinh\theta)+\frac{n}{2}\right], \tag{6}$$

taking $\tanh\theta = m\mathrm{i}/n$ to eliminate the term of $\left(\bar{b}^2-\hat{b}^{\dagger 2}\right)$ and substituting the value of m and n, then we get

$$\delta\hat{H}_1 = -\mathrm{i}\sqrt{\beta^2+(\chi\alpha_0)^2}\bar{b}\hat{b}-\frac{\mathrm{i}}{2}\sqrt{\beta^2+(\chi\alpha_0)^2}+\frac{\mathrm{i}}{2}\beta. \tag{7}$$

Here $\hat{b}$ and $\bar{b}$ are bosonic annihilation and creation operators that satisfy $\left[\hat{b},\bar{b}\right]=1$. It is important to note that $\hat{b}\neq\bar{b}^\dagger$ since θ is complex. The vacuum state of $\hat{b}$ bosons is defined via $\hat{b}|0\rangle=0$. It can be seen from Eq. (7) that the Hamiltonian has a negative imaginary part, which represents the dissipation of the eigenstate. And the larger its absolute value, the faster corresponding eigenstates dissipate, and therefore the shorter its lifetime. $|0\rangle$ is defined as the steady state because its eigenvalue has the largest negative imaginary part.

Considering the aforementioned Bogoliubov transformation and bosonic commutation relation, the following result can be obtained:

$$\begin{aligned}\hat{b}^\dagger &= \bar{b}\left(\left|\cosh\frac{\theta}{2}\right|^2-\left|\sinh\frac{\theta}{2}\right|^2\right)\\ &\quad+\hat{b}\left(-\mathrm{i}\sinh\frac{\theta}{2}\cosh^*\frac{\theta}{2}+\mathrm{i}\sinh^*\frac{\theta}{2}\cosh\frac{\theta}{2}\right),\\ \langle 0|\,\hat{b}\hat{b}^\dagger\,|0\rangle &= \left|\cosh\frac{\theta}{2}\right|^2-\left|\sinh\frac{\theta}{2}\right|^2.\end{aligned} \tag{8}$$

Therefore, we have

$$\begin{aligned}&\left\langle\delta\hat{d}_1^{\dagger 2}\right\rangle=\frac{1}{2}\frac{\chi\alpha_0}{\beta},\ \left\langle\delta\hat{d}_1^2\right\rangle=\frac{1}{2}\frac{\chi\alpha_0}{\beta},\\ &\left\langle\delta\hat{d}_1^\dagger\delta\hat{d}_1\right\rangle=\frac{1}{2}\left(\frac{\sqrt{\beta^2+(\chi\alpha_0)^2}}{\beta}-1\right),\\ &\left\langle\delta\hat{d}_1\delta\hat{d}_1^\dagger\right\rangle=\frac{1}{2}\left(\frac{\sqrt{\beta^2+(\chi\alpha_0)^2}}{\beta}+1\right).\end{aligned} \tag{9}$$

Taking into account the feasibly experiential observable, we introduce the amplitude quadrature operator $\delta\hat{X}_{d_1}=\delta\hat{d}_1+\delta\hat{d}_1^\dagger$ and phase quadrature operator $\delta\hat{P}_{d_1}=-\mathrm{i}\left(\delta\hat{d}_1-\delta\hat{d}_1^\dagger\right)$ whose quantum fluctuation can be expressed as

$$\begin{aligned}\left\langle\left|\delta\hat{X}_{d_1}\right|^2\right\rangle&=\frac{\chi\alpha_0}{\beta}+\frac{\sqrt{\beta^2+(\chi\alpha_0)^2}}{\beta},\\ \left\langle\left|\delta\hat{P}_{d_1}\right|^2\right\rangle&=-\frac{\chi\alpha_0}{\beta}+\frac{\sqrt{\beta^2+(\chi\alpha_0)^2}}{\beta}.\end{aligned} \tag{10}$$

3. Dissipative phase transition and the associated quantum correlations

In practice, the particle loss of the cavity modes is mainly due to the transmission of input and output mirrors. As nondegenerate optical parametric amplification (NOPA) processing is shown in Fig. 3,[33,34] it is assumed that the transmission loss rates of the output mirror to the three modes in the cavity are γ_0, γ_1, and γ_2, respectively. And the transmission loss rates of the input mirror to the three modes in the cavity are ρ_0, ρ_1, and ρ_2, respectively. As stated above, due to the nearly degenerated signal and idle modes, one can make the assumption that $\gamma_1=\gamma_2=\gamma$ and $\rho_1=\rho_2=\rho$. $\hat{b}_i^{\mathrm{in}}(t)$, $\hat{c}_i^{\mathrm{in}}(t)$ $(i=0,1,2)$ are the input of the vacuum field at the right and left end of the cavity, respectively.

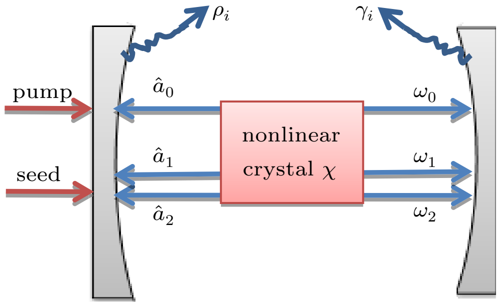

Fig. 3. Schematic of the optical parametric processing. Pump mode $\hat{a}_0$ with the frequency of ω_0, signal mode $\hat{a}_1$ with the frequency of ω_1 and idle mode $\hat{a}_2$ with the frequency of ω_2 are interactive in the cavity. γ_i and ρ_i $(i=0,1,2)$ are the transmission loss rates of the output and input mirror to the three modes in the cavity, respectively. χ is the nonlinear coefficient.

So the dissipative term is $-\mathrm{i}\left(\gamma_i+\rho_i\right)\hat{a}_i^\dagger\hat{a}_i$, Hamiltonian of the system is expressed as

$$\begin{aligned}H=&\sum_{i=0,1,2}\omega_i\hat{a}_i^\dagger\hat{a}_i+\mathrm{i}\chi\left(\hat{a}_0\hat{a}_1^\dagger\hat{a}_2^\dagger-\hat{a}_0^\dagger\hat{a}_1\hat{a}_2\right)\\ &+\mathrm{i}\left(\varepsilon_{\mathrm{p}}\hat{a}_0^\dagger-\varepsilon_{\mathrm{p}}^*\hat{a}_0\right)-\sum_{i=0,1,2}\mathrm{i}\left(\gamma_i+\rho_i\right)\hat{a}_i^\dagger\hat{a}_i.\end{aligned} \tag{11}$$

According to the quantum Langevin equation of motion of the NOPA cavity[33,34]

$$\begin{aligned}\dot{\hat{a}}_0&=-\mathrm{i}\left[\hat{a}_0,H\right]-\left(\gamma_0+\rho_0\right)\hat{a}_0+\sqrt{2\gamma_0}\hat{b}_0^{\mathrm{in}}+\sqrt{2\rho_0}\hat{c}_0^{\mathrm{in}},\\ \dot{\hat{a}}_1&=-\mathrm{i}\left[\hat{a}_1,H\right]-\left(\gamma+\rho\right)\hat{a}_1+\sqrt{2\gamma}\hat{b}_1^{\mathrm{in}}+\sqrt{2\rho}\hat{c}_1^{\mathrm{in}},\\ \dot{\hat{a}}_2&=-\mathrm{i}\left[\hat{a}_2,H\right]-\left(\gamma+\rho\right)\hat{a}_2+\sqrt{2\gamma}\hat{b}_2^{\mathrm{in}}+\sqrt{2\rho}\hat{c}_2^{\mathrm{in}}.\end{aligned} \tag{12}$$

Substituting $\hat{a}_i=a_i(t)\,\mathrm{e}^{-\mathrm{i}\omega_i t}$ $(i=0,1,2)$ into Eq. (12), we get

$$\begin{aligned}\dot{\hat{a}}_0(t)=&-\left(\gamma_0+\rho_0\right)\hat{a}_0(t)+\left|\varepsilon_{\mathrm{p}}\right|\mathrm{e}^{-\mathrm{i}\theta_0}-\chi\hat{a}_1(t)\,\hat{a}_2(t)\\ &+\sqrt{2\gamma_0}\hat{b}_0^{\mathrm{in}}(t)+\sqrt{2\rho_0}\hat{c}_0^{\mathrm{in}}(t),\\ \dot{\hat{a}}_1(t)=&-\left(\gamma+\rho\right)\hat{a}_1(t)+\chi\hat{a}_0(t)\,\hat{a}_2^\dagger(t)\\ &+\sqrt{2\gamma}\hat{b}_1^{\mathrm{in}}(t)+\sqrt{2\rho}\hat{c}_1^{\mathrm{in}}(t),\\ \dot{\hat{a}}_2(t)=&-\left(\gamma+\rho\right)\hat{a}_2(t)+\chi\hat{a}_0(t)\,\hat{a}_1^\dagger(t)\\ &+\sqrt{2\gamma}\hat{b}_2^{\mathrm{in}}(t)+\sqrt{2\rho}\hat{c}_2^{\mathrm{in}}(t).\end{aligned} \tag{13}$$

We assume the average value of cavity mode operators $\langle a_i(t)\rangle=\alpha_i\,\mathrm{e}^{-\mathrm{i}\theta_i}$ $(i=0,1,2)$. Under the condition of steady state, Eq. (14) is obtained because the average values of vacuum field operators are equal to 1,

$$0=-\left(\gamma_0+\rho_0\right)\alpha_0\,\mathrm{e}^{-\mathrm{i}\theta_0}+\left|\varepsilon_{\mathrm{p}}\right|\mathrm{e}^{-\mathrm{i}\theta_0}-\chi\alpha_1\alpha_2\,\mathrm{e}^{-\mathrm{i}\theta_0},$$

$$
\begin{aligned}
0 &= -(\gamma+\rho)\,\alpha_1\,\mathrm{e}^{-\mathrm{i}\theta_1}+\chi\alpha_0\alpha_2\,\mathrm{e}^{-\mathrm{i}\theta_1},\\
0 &= -(\gamma+\rho)\,\alpha_2\,\mathrm{e}^{-\mathrm{i}\theta_2}+\chi\alpha_0\alpha_1\,\mathrm{e}^{-\mathrm{i}\theta_2}.
\end{aligned}
\tag{14}
$$

Hence, the solutions of steady state are as follows:

$$
\begin{aligned}
&\alpha_0=(\gamma+\rho)/\chi,\\
&\alpha_1=\alpha_2=\sqrt{\varepsilon_\mathrm{p}-(\gamma_0+\rho_0)\,(\gamma+\rho)/\chi}/\chi, \qquad (15\mathrm{a})\\
&\alpha_0=\varepsilon_\mathrm{p}/(\gamma_0+\rho_0),\quad \alpha_1=\alpha_2=0. \qquad (15\mathrm{b})
\end{aligned}
$$

According to Eq. (15), only when $\varepsilon_\mathrm{p}-(\gamma_0+\rho_0)\,(\gamma+\rho)/\chi>0$, $\alpha_1=\alpha_2\neq 0$ is true. That is to say, when $\varepsilon_\mathrm{p}>\varepsilon^\mathrm{thres}$ (threshold: $\varepsilon^\mathrm{thres}=(\gamma_0+\rho_0)\,(\gamma+\rho)/\chi)$, the steadystate solutions above the threshold are Eq. (15a), and the signal mode and idle mode oscillate in the cavity where the coherent interaction is dominant. Otherwise, the steady-state solutions below the threshold are Eq. (15b), and the signal mode and idle mode will be in the vacuum field ($\alpha_1=\alpha_2=0$) where the loss is dominant.

As expected, a dissipative phase transition (second-order phase transition) occurs around the threshold of pump light in such a system. Figure 4(a) shows order parameters α_i $(i=1,2)$ as a function of the intensity of the driving pump light ε_p. We introduce $k=\chi\alpha_0/(\gamma+\rho)$. If the pump power is below the threshold, $k=\varepsilon_\mathrm{p}/\varepsilon^\mathrm{thres}$ and $0\le k<1$ are obtained by the solution of steady state. If the pump power is above the threshold, then one gets $k=1$.

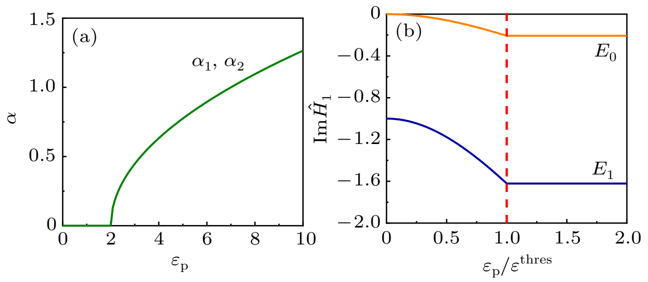

Fig. 4. Dissipative phase transition in the open quantum optical system. (a) Order parameters α_i $(i=1,2)$ for $\gamma_0+\rho_0=10$, $\gamma+\rho=1$, $\chi=5$, and $\varepsilon^\mathrm{thres}=2$ as a function of the intensity of the driving pump light ε_p. (b) The imaginary parts of eigenvalues E_0 (brown curve) and E_1 (blue curve) as a function of $\varepsilon_\mathrm{p}/\varepsilon^\mathrm{thres}$. The red dashed line shows the position of the threshold, the so-called critical point.

The dissipative coefficient $\gamma+\rho$ here corresponds to β of part 2. According to Eq. (7), the Hamiltonian with the second-order quantum fluctuation term can be rewritten as

$$
\delta\hat{H}_1=\left[-\mathrm{i}\sqrt{1+k^2}\bar{b}b-\frac{\mathrm{i}}{2}\left(\sqrt{1+k^2}-1\right)\right]\Big/(\gamma+\rho). \tag{16}
$$

As an example, we look at the eigenvalues E_0 and E_1 corresponding to $\bar{b}b=0$ and 1, respectively. Figure 4(b) shows the imaginary parts of E_0 (brown line) and E_1 (blue line) in the unit of $\gamma+\rho$ as a function of $\varepsilon_\mathrm{p}/\varepsilon^\mathrm{thres}$, respectively. The part of $0\le\varepsilon_\mathrm{p}/\varepsilon^\mathrm{thres}<1$ corresponds to the situation below the threshold, and the part of $\varepsilon_\mathrm{p}/\varepsilon^\mathrm{thres}>1$ corresponds to that above the threshold, indicated by the red dashed line in the figure. Strikingly, there also exists the non-analytical behavior for both curves in the threshold, the critical point. Below the threshold, the higher the pump power is, the faster the eigenstate dissipates. Above the threshold, as the pump power continues to increase, the dissipation will tend to saturation. In terms of the order parameters and the eigenvalues of the non-Hermitian Hamiltonian, it indicates that a dissipative phase transition (second-order phase transition) occurs around the threshold of pump light in such a system.

We proceed to discuss the quantum correlation in terms of squeezing. Figure 5 shows $\langle|\delta\hat{X}_{d_1}|^2\rangle$ and $\langle|\delta\hat{P}_{d_1}|^2\rangle$ as a function of $\varepsilon_\mathrm{p}/\varepsilon^\mathrm{thres}$, indicated by the green and purple solid curves, respectively, and the black solid line represents SNL. Below the threshold, the noise of the amplitude quadrature operator is raised above the SNL, while the noise of the phase quadrature operator is squeezed below the SNL. And by increasing the intensity of pump light to approach the threshold, the nonclassical optical field with a higher squeezing degree can be achieved. However, above the threshold, with the increase of pump power, the squeezing degree tends to saturation. In terms of the quantum correlation, it similarly indicates that a dissipative phase transition (second-order phase transition) occurs around the threshold of pump light.

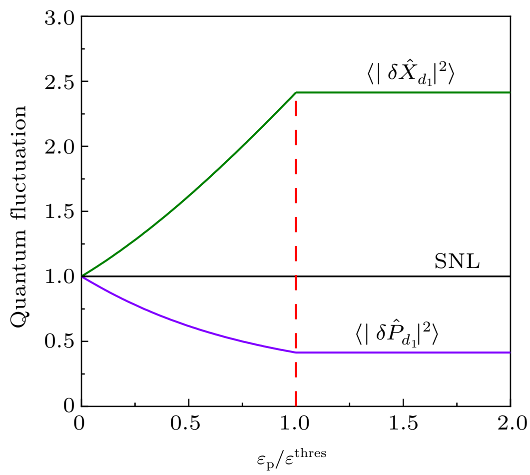

Fig. 5. The green and purple solid curves represent quantum fluctuation of $\delta\hat{X}_{d_1}$ and $\delta\hat{P}_{d_1}$ as a function of $\varepsilon_\mathrm{p}/\varepsilon^\mathrm{thres}$, respectively. The red dashed line shows the position of the threshold, the so-called critical point. The black solid line represents SNL.

In summary, the existence of dissipative term leads to the threshold to be non-zero, which leads to the second-order phase transition of the order parameters, the eigenvalues of non-Hermitian Hamiltonian and the quantum correlation near the threshold. And the larger the dissipation is, the larger the threshold value will be. Then the intensity of pump light is needed to be bigger than the threshold value to eliminate the domination of dissipation and make the signal mode and idle mode oscillate in the cavity. Otherwise, the signal mode and idle mode will be in the vacuum field because their steady-state solution is zero below the threshold. That is to say, the dissipative phase transition is attributed to the competition between the coherent interaction and dissipation.

4. Conclusion and perspective

We report the application of non-Hermitian theory in an open quantum optical system. Due to the competition between coherent nonlinear interaction and the dissipation induced by the system loss, a dissipative phase transition is observed. Moreover, we analyze the quantum correlations in terms of squeezing around the critical point. Our work provides new insight into the quantum simulation of the non-Hermitian quantum system, especially the quantum properties.

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