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ABSTRACT

Preparation of atomic Bose–Einstein condensate (BEC) with tunable interactions in a quasi-one-dimensional (quasi-1D) optical trap is essential for both the observation of bright matter-wave solitons and the quantum simulation based on the discrete atomic momentum states. However, the quasi-1D BEC has been obtained by a complex process, which includes the creation of a three-dimensional BEC and its adiabatic transform into a quasi-1D trap. Here, we report the direct creation of a quasi-1D BEC of ^{133}Cs atoms by the fast evaporative cooling of ultracold atoms prepared by the degenerated Raman sideband cooling. We produce the pure BEC of up to 5.5×10^4 atoms in a quasi-1D optical trap with an evaporative time of 6 s. We demonstrate the anisotropic expansion of the atomic cloud after the release from the quasi-1D trap and study the dependence of both the phase space density and the temperature on the number of atoms in the trap during the evaporative cooling. Our study facilitates the promising applications of quasi-1D interacting atomic gases.

I. INTRODUCTION

Quantum degenerate atomic gases provide a versatile platform for many significant fundamental and applied studies ranging from quantum simulations of many-body physics to precision measurements with highly controllable experimental conditions.[1–4] For ultracold atoms in a quasi-one-dimensional (quasi-1D) optical trap, bright matter-wave solitons were created by quenching the atomic interaction from repulsive to attractive. The momentum lattices based on the laser-coupled discrete atomic momentum states of quasi-1D BEC have been used to engineer complex Hamiltonians,[5] enabling the observation of many exotic phenomena, including the mobility edge in the quasi-periodic Aubry–André model,[6–8] the topological soliton in the Su–Schrieffer–Heeger model[9] and the dissipative Aharonov–Bohm transport in the non-Hermitian model.[10] In addition, the chiral dynamics of atomic population was explored with tunable artificial gauge fields, which were realized in the square and triangular ladders in momentum lattices.[11–13]

To prepare the atomic BEC in a quasi-1D optical trap, it needs to transfer the three-dimensional (3D) BEC obtained by the general evaporative cooling into an optical trap with quasi-1D geometry.[14–16] For avoiding the substantial loss and heating of atoms during the transfer procedure, an adiabatic transfer with the sufficiently slow change for the trap potential is an indispensable, however, time-consuming step. Therefore, it is of fundamental importance for the fast creation of atomic BEC in a quasi-1D optical trap to facilitate the studies of both the quantum simulation based on momentum lattices of ultracold atoms[7–13] and the formation of nonlinear matter-wave solitons.[17–21]

Here, we demonstrate the direct creation of quasi-1D BEC of ^{133}Cs atoms via the fast evaporative cooling for ultracold atoms, which is prepared through the 3D degenerated Raman sideband

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cooling. The evaporative cooling is effectively implemented by reducing the magnetic field gradient and the powers of several optical trap laser beams, while the power of one trap laser beam is increased to provide the strongly radial confinement for the quasi-1D optical trap. We observe and analyze the anisotropic expansion of the atomic cloud after the release from the quasi-1D trap due to the large imbalance between the radial and axial trap frequencies. We study the dependence of the phase space density (PSD) and temperature on the number of atoms during the cooling procedure, and the experimental data are in good agreement with the equation model based on the scaling laws for evaporative cooling.

II. DIRECT EVAPORATIVE COOLING TO QUASI-1D BEC

We apply the 3D degenerated Raman sideband cooling to the cold ^{133}Cs atoms confined in a magneto-optical trap and obtain about 1×10^7 atoms in the hyperfine ground state $|6S_{1/2}; F = 3, m_F = 3\rangle$

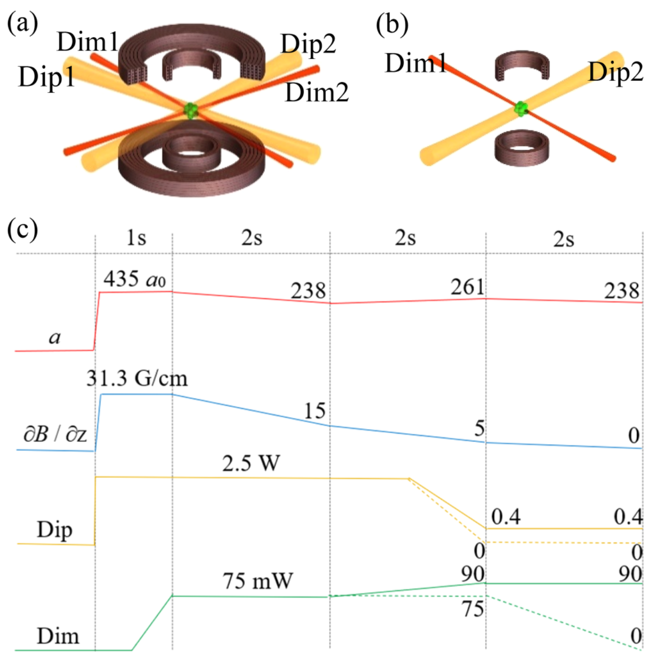

FIG. 1. Schematic of the experimental setup and time sequence of evaporative cooling. (a) Experimental geometry and configuration of both laser beams and magnetic coil pairs. The combination of a crossed dipole trap (Dip1 and Dip2) with the magnetic field gradient $\partial B/\partial z$ produced by a pair of the outer magnetic coils pair is used to load cold ^{133}Cs atoms into a magnetically levitated dipole trap. The dimple trap (Dim1 and Dim2) is added to largely increase the atomic density before the evaporative cooling. The atomic scattering length is tuned by the uniform magnetic field, which is produced by a pair of the inner magnetic coils. (b) A quasi-1D optical trap is composed of the dimple trap laser beam (Dim1) and the dipole trap laser beam (Dip2) with an intersection angle of $\sim 90°$. (c) The time sequence of fast evaporative cooling for the direct production of quasi-1D atomic BEC. The power of the dimple trap laser beam (Dim1) is increased to form the strongly radial confinement of quasi-1D trap during the evaporative process.

with the temperature of $T \sim 1\,\mu$K.[22,23] The ultracold atoms are subsequently loaded into a magnetically levitated crossed optical dipole trap, which consists of a vertical magnetic field gradient of $\partial B/\partial z = 31.3$ G/cm and two horizontally crossed 1064-nm laser beams with an intersection angle of $\sim 90°$,[24] as shown in Fig. 1(a). These two dipole laser beams (Dip1 and Dip2) have the same radius of $\sim 300\,\mu$m and power of 2.5 W. To enhance the atomic density before the evaporative cooling, we add a tight dimple trap, which is composed of two 1064-nm laser beams (Dim1 and Dim2) with the almost same radius of $\sim 58\,\mu$m and the same power of 75 mW, in the center of the crossed dipole trap.[25–31] Each dimple laser beam has an intersection angle $\sim 12°$ with its neighboring dipole trap laser beam. The trap frequencies on the bottom of the combined potential are $(\omega_x, \omega_y, \omega_z) = 2\pi \times (131, 131, 186)$ Hz.

The uniform magnetic field is used to tune the atomic s-wave scattering length to $a = 453\ a_0$ not only for the quick thermal equilibrium after the loading of the magnetically levitated optical trap but also for avoiding the large three-body loss of atoms during the loading of dimple trap,[32] where a_0 is the Bohr radius. At this point, the number of atoms is $N = 2.6 \times 10^6$ with the temperature of $T = 3.4\,\mu$K, and the peak atomic density increases to $n = 1.8 \times 10^{13}$ cm^{-3}. The PSD is given as $D = 0.06$. Here, the PSD is defined as $D = n\lambda_{dB}^3$ with the peak atomic density of $n = N\varpi(m\lambda_{dB}/h)^3$ (ϖ the geometric mean of trap frequencies and m the mass of ^{133}Cs atoms) and the thermal de Broglie wavelength $\lambda_{dB} = h(2\pi m k_B T)^{-1/2}$ (k_B the Boltzmann constant and h the Planck constant).[33] Afterward, we perform the efficient evaporative cooling of atoms, and the corresponding time sequence is shown in Fig. 1(c).

The initial evaporative cooling is implemented by linearly reducing the magnetic field gradient from $\partial B/\partial z = 31.3$ to 15 G/cm in 2 s, during which the trap depth is reduced by tilting the trap potential in the vertical direction. The scattering length is tuned from $a = 435\ a_0$ to $238\ a_0$, and the three-body recombination rate is slowly reduced to its minimum. The atomic temperature drops to $T = 530$ nK, but the density and PSD are increased to $n = 5.2 \times 10^{13}$ cm^{-3} and $D = 0.47$, respectively. Subsequently, the magnetic field gradient is reduced to $\partial B/\partial z = 5$ G/cm in 2 s, and the scattering length is slightly increased to $a = 261\ a_0$ for the optimum cooling. The power of one dipole trap laser beam (Dip1) is rapidly reduced to 0 in 1 s, and the power of the other dipole trap laser beam (Dip2) is reduced to 0.4 W. To directly obtain the atomic BEC in a quasi-1D trap, we increase the power of one dimple trap laser beam (Dim1) to 90 mW in 2 s for providing strongly radial confinement.

The final evaporative cooling is performed by reducing the magnetic field gradient to 0 in 2 s, and the scattering length is back to $a = 238\ a_0$. The optical evaporative cooling is implemented by linearly lowering the power of the dimple trap laser beam (Dim2) from 75 mW to 0. As a result, we produce a pure BEC with 5.5×10^4 atoms in the quasi-1D optical trap, which consists of a dimple trap laser beam (Dim1) and a dipole trap laser beam (Dip2) with the crossed angle of $\sim 90°$ as shown in Fig. 1(b). The resulting trap frequencies are $(\omega_x, \omega_y, \omega_z) = 2\pi \times (11, 126, 98)$ Hz, which are measured by monitoring the motion of the kicked atoms in the trap. The PSD is estimated as $D = 12$ when considering the condensation in the Thomas–Fermi regime.

Different from the isotropic expansion of thermal atoms in the time-of-flight (TOF) measurement, the density distribution of the expanding atomic BEC released from a quasi-1D trap is anisotropic in the Thomas–Fermi regime.[34] After the end of evaporative cooling, we switch off the trap to release the BEC, and the uniform magnetic field is rapidly tuned to 17 G for the zero-crossing scattering length.[32] In Fig. 2(a), we show the TOF images of atomic BEC and observe that the condensate expands much faster in the radial than in the axial directions, where the x-direction along the remaining dimple laser beam (Dim1) corresponds to the axial direction. The optical density distributions in the z-direction extracted from the images for different flight times in Fig. 2(a) are shown in Fig. 2(b), and the distribution width of the atomic cloud increases with time.

To quantitatively describe the anisotropic expansion of the atomic BEC released from the quasi-1D trap, we plot the $1/e$ widths of atomic cloud σ_x and σ_z in the axial and radial directions as a function of flight time t in Fig. 3. Compared to the almost constant σ_x for $t \leq 24$ ms, σ_z shows a fast increase with t. This dramatic difference exhibited in the evolutions of σ_x and σ_z in the TOF process demonstrates the uniquely anisotropic expansion of quasi-1D atomic BEC, where the anisotropic configuration of quasi-1D trap leads to the distinct velocity of atoms in the radial and axial directions. The effective temperature of quasi-1D atomic BEC can be estimated by fitting the data by using the formula

$$\sigma_{i,t} = \sqrt{\sigma_{i,0}^2 + \frac{k_B T_i}{m} t^2}, \quad (1)$$

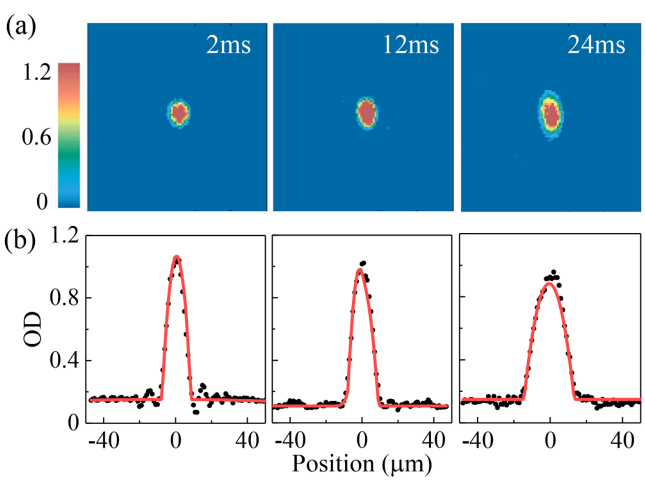

FIG. 2. Time-of-flight images of expanding BEC after the release from the quasi-1D trap. (a) The images are taken at the zero-crossing scattering length near the magnetic field of 17 G for different flight times. The BEC expands much faster in the radial than the axial directions due to the geometric anisotropy of quasi-1D trap. (b) Single-line optical density (OD) profiles in the radial (or vertical) direction are extracted from the images in (a), and the distribution width increases with the flight time. The red curves represent the fit with the density distribution in the Thomas–Fermi regime.

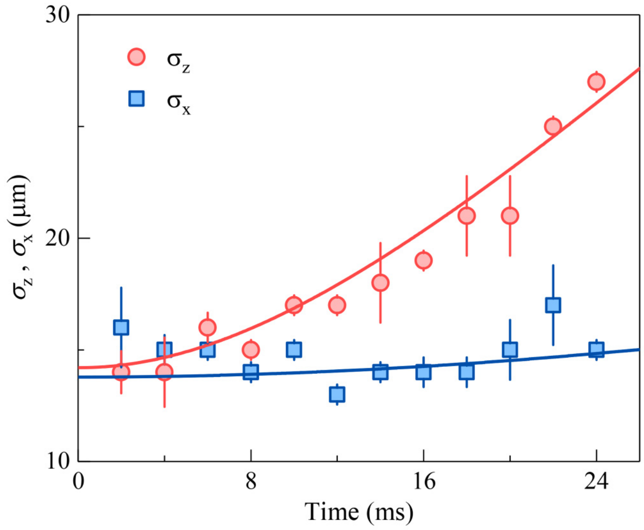

FIG. 3. The $1/e$ width of the atomic cloud released from the quasi-1D trap as a function of flight time. σ_x and σ_z denote the widths of the atomic cloud in the axial and radial directions, respectively. The atomic cloud shows a fast expansion in the radial direction in compared to the almost constant size in the axial direction. The solid curves are obtained by fitting the experimental data using Eq. (1). All error bars denote standard errors.

where subscript i represents the x or z axis, and $\sigma_{i,t}$ is the $1/e$ width of atom cloud at the time t.[35] The effective temperatures are $T_x = 0.85(14)$ nK and $T_z = 13.31(11)$ nK, the lower temperature in the axial direction rather than that in the radial direction is related to the shallow trap in the axial direction for the quasi-1D trap at the end of evaporative cooling.

III. DEPENDENCE OF PSD AND TEMPERATURE ON ATOM NUMBER

We further evaluate the efficiency of the fast evaporative cooling that is used to directly obtain the quasi-1D BEC. The mean evaporation efficiency is given as $\overline{\gamma} = -\ln(D/D_0)/\ln(N/N_0)$, where D_0 and N_0 are the PSD and the atom number after the loading of the crossed dimple trap.[36] By averaging the efficiencies in the three evaporative steps, we obtain the mean evaporative efficiency as $\overline{\gamma} = 3.08 \pm 0.12$. The variation of the PSD with the atom number during the evaporative cooling is shown in Fig. 4(a), and the PSD increases with the reduction of atom number. The numerical simulation based on the calculated $\overline{\gamma}$ is in good agreement with the experimental data. Although the power of one dimple trap laser beam (Dim1) is increased to largely enhance the radial confinement during the evaporative process, we find that the evaporative efficiency is not significantly affected by the formation of quasi-1D trap in the evaporation process.

In Fig. 4(b), we show the variation of the average temperature with the atom number, and the evaporative cooling leads to the reduction of both the atomic temperature and the atom number. The error of the atomic temperature comes from the model

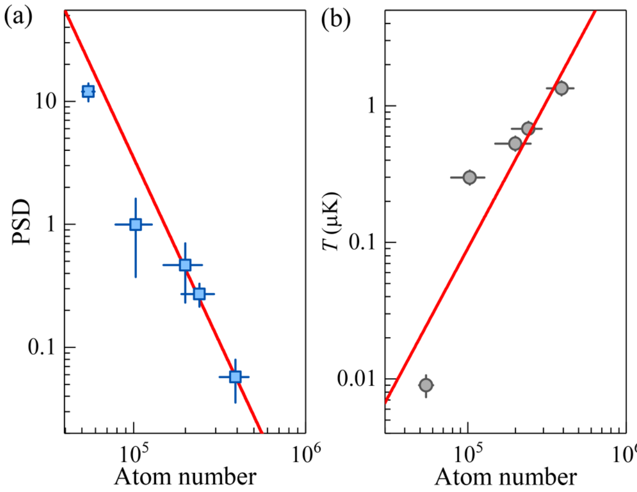

FIG. 4. Variations of the PSD in (a) and the average temperature in (b) during the evaporative cooling as a function of the atom number. The solid curve in (a) is the numerical simulation based on the mean evaporative efficiency $\overline{\gamma} = 3.08 \pm 0.12$, which is obtained by averaging the efficiencies in all evaporative steps. The solid curve in (b) is obtained by using the power-law dependence of temperature on the atom number to fit the experimental data. All error bars denote standard errors.

uncertainty and the fitting error. We analyze the dependence of temperature on the atom number by using a power-law relation as $T \propto N^{\alpha}$. Here, α is used to parameterize the cooling efficiency by removing the atoms from the trap, and it is defined as

$$\alpha = \frac{N}{T}\frac{dT}{dN} = \frac{\eta' - 3}{3 - \nu}, \quad (2)$$

where $\eta' = \eta + (\eta - 5)/(\eta - 4)$. In our evaporative process, the mean truncation parameter is given as $\eta = \langle U/k_B T\rangle = 8.05$, and U is the depth of trap potential in the vertical direction. Parameter ν can be obtained by using $\overline{\varpi} \propto U^{\nu}$ to fit the dependence of the mean trap frequency $\overline{\varpi}$ on the trap depth U. When substitute the parameters η and ν into Eq. (2), we obtain $\alpha = 2.16$. We use the power-law relation to fit the observed dependence of temperature on the atom number, and the fitting curve is in qualitative agreement with the experimental data.

IV. CONCLUSION

We have demonstrated a fast evaporative cooling approach for the direct production of interacting quasi-1D atomic BEC. The anisotropic expansion has been observed for the atomic BEC released from the quasi-1D trap. Different from the previously long-time method by combining the preparation of three-dimensional BEC and its adiabatic transfer into a quasi-1D trap, we have increased the power of one trap laser beam to form the quasi-1D trap during the evaporative process, which enables the direct production of quasi-1D BEC via a fast evaporative cooling. Moreover, the dependence of the PSD and temperature on the number of atoms in the trap illustrates that the formation of quasi-1D trap with the increased laser power has little impact on the efficiency of the fast evaporative cooling. The direct production of interacting quasi-1D BEC through fast evaporation has significant applications in the studies of both the quantum simulations in momentum lattices and the nonlinear matter-wave solitons.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

All authors discussed the results, contributed to the data analysis and co-wrote the manuscript.

Huiying Du: Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Yuqing Li:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Yunfei Wang:** Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Writing – original draft (equal); Writing – review & editing (equal). **Jizhou Wu:** Investigation (equal); Writing – original draft (equal); Writing – review & editing (equal). **Wenliang Liu:** Writing – original draft (equal); Writing – review & editing (equal). **Peng Li:** Writing – original draft (equal); Writing – review & editing (equal). **Yongming Fu:** Writing – original draft (equal); Writing – review & editing (equal). **Jie Ma:** Data curation (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Liantuan Xiao:** Resources (equal); Supervision (equal); Validation (equal). **Suotang Jia:** Funding acquisition (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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