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Narrow laser linewidth measurement with the optimal demodulated Lorentzian spectrum

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A method called the optimal demodulated Lorentzian spectrum is employed to precisely quantify the narrowness of a laser's linewidth. This technique relies on the coherent envelope demodulation of a spectrum obtained through short delayed self-heterodyne interferometry. Specifically, we exploit the periodic features within the coherence envelope spectrum to ascertain the delay time of the optical fiber. Furthermore, the disparity in contrast within the coherence envelope spectrum serves as a basis for estimating the laser's linewidth. By creating a plot of the coefficient of determination for the demodulated Lorentzian spectrum fitting in relation to the estimated linewidth values, we identify the existence of an optimal Lorentzian spectrum. The corresponding laser linewidth found closest to the true value is deemed optimal. This method holds particular significance for accurately measuring the linewidth of lasers characterized as narrow or ultranarrow.

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1. INTRODUCTION

Narrow linewidth lasers characterized by high coherence and minimal noise are essential for various applications, including optical atomic clocks [1], distributed fiber sensing [2], precise quantum manipulation of atoms and molecules [3,4], gravitational wave detection [5], and other fields [6]. Consequently, a precise measurement of laser linewidth is a fundamental requirement for these applications, attracting significant attention [7]. Traditional methods for measuring laser linewidth, such as optical spectrometers and Fabry–Perot interferometers, are limited in terms of spectral resolution. Currently, commercial optical spectrometers based on diffraction gratings afford resolutions of approximately 0.01 nm (equivalent to the GHz level at the commonly used near-infrared wavelengths), whereas Fabry–Perot scanning interferometers can typically achieve resolutions in the MHz range. However, lasers with linewidths as narrow as kHz or even sub-Hz levels have been realized in various applications [6]. Conventional measurement methods cannot accurately measure such narrow laser linewidths.

The heterodyne beat method can provide high measuring precision for laser linewidth, which is a Lorentzian lineshape pointed out by Scully and Lamb [8]. However, this method requires an extra laser with very narrow linewidth and close frequency to the laser under test, which is hard to be achieved. In contrast, the self-heterodyne method needs no extra laser and can get linewidth values from the beat signal of beams arising from the same laser under test. Thus, it is widely used for the ultranarrow linewidth laser [9]. There mainly are two ways when utilizing the self-heterodyne method for linewidth measurement: directly calculating the laser linewidth using the power spectrum density (PSD) of the laser and deducing the linewidth indirectly based on the relationship between the phase noise and linewidth [7]. Although the latter also involves phase and frequency noises, the calculated processes involving b-separation line theory are very complicated [10,11]. Alternatively, the power spectrum contains more intuitive linewidth information, and it is relatively easy to obtain. In many aspects of applications based on ultranarrow lasers, the linewidth characteristic measurement is enough. Therefore, a large proportion of linewidth measurement experiments focus on the former. In 1980 Okoshi *et al.* [12] first proposed a delayed self-heterodyne interferometry (DSHI) method to measure linewidth. It involves splitting a single laser beam into two, with a controlled time delay between them, followed by recording the power spectral density (PSD) of the resulting beat signal to deduce the laser linewidth via Lorentzian fitting. To ensure that the beat signal exhibits a Lorentzian lineshape, the delay time must be at least six times greater than the coherence time of the laser being analyzed [13]. For lasers with sub-kHz linewidths, this requires delay fibers exceeding 1000 km in length. However, the use of such long fibers is economically burdensome, results in significant optical power loss, and broadens the measured spectrum due to $1/f$ frequency noise [13,14].

Reducing the length of the delay fiber effectively eliminates the impact of $1/f$ noise. Thus, a method known as short delayed self-heterodyne interferometry (SDSHI) [13,15] has been proposed for determining laser linewidth. In this approach, a coherence envelope is observed that combines a Lorentzian spectrum with a periodic modulation spectrum. Recently, an increasing number of studies have focused on the derivation of laser linewidth from the measured PSD of SDSHI.

Two primary approaches exist for determining laser linewidth from the coherence envelope of the SDSHI spectrum. One approach involves formula calculations using specific points from the coherent envelope spectrum. This method relies on comparing the amplitude difference of the coherent envelope (ADCCE) with theoretical calculations that are related to and yield the laser linewidth. ADCCE can involve metrics such as the contrast difference between the second peak and second valley (CDSPSV) [16,17], dual-parameter acquisition (DPA) [18], or the amplitude difference between any adjacent pair of extreme points [19]. These methods require manual measurement of amplitude differences within the coherent envelope and impose strict requirements on the length of the optical fiber to avoid noise floor interference with short fibers or $1/f$ noise with long fibers [18,19].

An alternative method is the coherent envelope demodulation (CEDM) spectrum, which is particularly suited for measuring narrow and ultranarrow linewidths. The concept involves recovering the Lorentzian spectrum through demodulation of the SDSHI spectrum and subsequently deriving the linewidth through Lorentzian fitting. He *et al.* [20] initially proposed the CEDM method, incorporating an iterative algorithm for measuring kHz-level linewidth lasers. The linewidth obtained from the SDSHI spectrum fitting was used as an initial estimate, demodulating the concealed Lorentzian spectrum. An updated linewidth derived from this spectrum served as a new estimate; this process was iterated until the tentative linewidth and derived linewidth converged. More recently, Xue *et al.* [21] and Bai *et al.* [22] respectively suggested setting the initial linewidth based on measurements with long delay fibers or CDSPSV. Regardless of the specific method employed, the iterative algorithm, with consistent convergence criteria, may introduce demodulation errors due to the random characteristics of the initial laser linewidth and repeated iterations.

Hence, in this paper, we propose an alternative approach for obtaining the CEDM spectrum in the measurement of narrow laser linewidths. Our method relies on fitting the derived CEDM spectrum with the Lorentzian formula itself, obviating the need for special initial laser linewidth estimates and repeated iterative calculations. Our approach hinges on the principle that the derived Lorentzian spectrum from the CEDM method achieves the best fit with the Lorentzian formula when the estimated linewidth closely aligns with the actual value. We quantify the degree of Lorentzian fitting using coefficients of determination obtained from the demodulated spectrum. Experimental results demonstrate the existence of a maximum value when plotting coefficients of determination against a range of estimated linewidth values. Consequently, the optimal demodulated Lorentzian spectrum (ODLS) and its corresponding linewidth are identified at the point of best fit. The ODLS method we present offers a straightforward and universally applicable approach for measuring narrow laser linewidths.

2. PRINCIPLE

The SDSHI spectrum serves as the cornerstone of the CEDM method for laser linewidth measurement. The principle underlying the SDSHI spectrum revolves around the partially coherent optical interference of beat signals.

The corresponding power spectrum function, S, can be expressed as follows [13,18,20,23]:

$$S(f, \Delta f) = S_1 S_2 + S_3. \tag{1}$$

Here,

$$S_1 = \frac{P_0^2}{4\pi} \frac{\Delta f}{\Delta f^2 + f^2}, \tag{2}$$

$$S_2 = 1 - \exp(-2\pi \Delta f \tau_d) \times \left[\cos(2\pi f \tau_d) + \Delta f \frac{\sin(2\pi f \tau_d)}{f}\right], \tag{3}$$

$$S_3 = \frac{\pi P_0^2}{2} \exp(-2\pi \Delta f \tau_d)\delta(f), \tag{4}$$

where P_0 is the power of the beat signal, Δf is the laser linewidth, $f = f_1 \pm f_0$ is the relative frequency of the beat signal (f_1 and f_0 are the frequency of beat signal and the radio frequency driving AOM, respectively), τ_d is the fiber delay time of one path with respect to the other path. The delayed fiber length, L, and the refractive index of the fiber core, n, are determined based on the expression $\tau_d = nL/c$ (c is the speed of light in vacuum). $\delta(f)$ is the impulse function.

From the expressions in Eqs. (2)–(4), the total power spectrum S in Eq. (1) is the δ-peak spectrum S_3 plus the product of the Lorentzian spectrum S_1 and the periodic modulation spectrum S_2. From Eq. (4), when $f_1 \neq f_0$, $\delta(f) = 0$, and thus, $S_3 = 0$; when $f_1 = f_0$, $S_3 =$ infinite. Therefore, we can ignore the item S_3 as the detected power spectrum is unstable at $f_1 = f_0$; thus, Eq. (1) can be simplified as follows:

$$S(f, \Delta f) = S_1 S_2. \tag{5}$$

As described in Section 1 (Introduction), two distinct approaches can be adopted for treating the SDSHI spectrum [shown with Eq. (5)] to derive the laser linewidth (Δf). These methods involve either calculations using specific points within the coherent envelope profile or the employment of Lorentzian fitting with the CEDM spectrum.

In the former method, ADCCE is commonly used owing to its simplicity and widespread applicability. Typically, this approach involves selecting a specific order, ΔS, and utilizing the contrast difference between peaks and valleys (CDPV) at that order to derive the laser linewidth. It can be conveniently expressed in a logarithmic coordinate system:

$$\Delta S(\Delta f) = 10\log_{10} S_{\text{peak}} - 10\log_{10} S_{\text{valley}}$$
$$= 10\log_{10} S\left(\frac{2L-1}{2\tau_d}, \Delta f\right) - 10\log_{10} S\left(\frac{m}{\tau_d}, \Delta f\right)$$
$$= 10\log_{10}\left(\frac{\Delta f^2 + (m/\tau_d)^2}{\Delta f^2 + ((2L-1)/2\tau_d)^2} \cdot \frac{1+\exp(-2\pi\Delta f\tau_d)}{1-\exp(-2\pi\Delta f\tau_d)}\right). \tag{6}$$

Here, the parameters $l = 2, 3, 4, \ldots$ and $m = 1, 2, 3, \ldots$ represent the order numbers of the peaks at positions within the l series and the valleys at positions within the m series, respectively. Owing to the large disturbance near the spectrum center and the influence from the noise floor at high orders [23], the contrast difference between the second peak and the second valley (CDSPSV) is typically chosen as $l = m = 2$ to derive the linewidth. As per Eq. (6), the exact value of τ_d and the values of order numbers l and m are required to derive linewidth Δf. Previous studies [16,20] have used the known length of optical fiber (L) and the refractive index of the fiber core (n) to calculate the value of τ_d. Thus, this method may introduce deviations from real values. Moreover, a lack of data for these two parameters is challenging. Therefore, we propose a more accurate and straightforward approach, where the value of τ_d can be directly derived from the periodic envelope spectrum [Eq. (5)] with a period of $1/\tau_d$ determined by Eq. (3).

The CEDM method also stems from Eq. (5). The core principle of CEDM is the exact recovery of the Lorentzian spectrum (S_1) containing the expected Δf via demodulation:

$$S_1 = \frac{S}{S_2}. \tag{7}$$

In this equation, S represents the modulated signal that can be readily measured in the experiment. The modulation signal S_2 encompasses two parameters: one is the fiber delay time (τ_d), as elaborated in the preceding paragraph, and the other is the linewidth (Δf), a crucial parameter derived from spectroscopic fitting with S_1. Previous studies [20–22] tackled this challenge using iterative algorithms to deduce the linewidth. They initiated the process with an estimated linewidth (Δf_{upd}) that was substituted into S_2. Subsequently, S_1 was fitted with the Lorentzian formula to yield an updated linewidth (denoted as Δf_{upd}). This updated linewidth then served as the new estimated value, iterating the procedure until Δf_{est} closely matched Δf_{upd} consistently.

Here, we introduce an alternative approach to determine the linewidth. The concept hinges on the premise that the derived spectrum S_1 exhibits the best fit with the Lorentzian formula when the estimated linewidth (Δf_{est}) closely approximates the actual value. In practical operations, we must plot a quantitative parameter (such as the coefficient of determination utilized in this study) that reflects the degree of conformity between the Lorentzian formula and the derived spectrum. This parameter should be plotted as a function of the estimated linewidth, enabling us to identify the best fit based on this relationship. Notably, the range of estimated linewidths should encompass extreme values for comprehensive analysis.

3. EXPERIMENTAL SETUP

Figure 1(a) illustrates our experimental setup. The output of a semiconductor laser with grating external cavity feedback (Toptica, DLC Pro), operating at a wavelength of 1557 nm, is split into two beams using a half-wave plate ($\lambda/2$) and a polarization beam splitter (PBS). One of these beams is coupled into an optical fiber for laser frequency stabilization based on the Pound–Drever–Hall (PDH) technique, while the other is coupled into a different fiber for laser linewidth measurement using the SDSHI spectrum method.

In the module dedicated to laser frequency stabilization, the output beam from the fiber is shaped by two lenses with focal lengths of 100 mm and −100 mm to facilitate coupling with an ultra-stable optical cavity. Adjusting the distance between these lenses enables control of the waist of the Gaussian beam. For our experimental parameters (laser wavelength of 1557 nm, cavity length of 10 cm, and radius of curvature of the concave mirror of 50 cm), the expected waist of the Gaussian beam $\omega_0 = 314.9$ μm and the measured value is 315 μm with an uncertainty of 1 μm, shown in Fig. 1(b).

To obtain the error signal corresponding to the cavity mode, a sinusoidal signal from the PDD110 module modulates the laser current through the MOD AC port. The modulated reflected beam is detected by PD2, and the electrical signal is directed to the input of PDD110 for frequency demodulation. The resulting demodulated error signal is presented in Fig. 1(c). This error signal is subsequently fed into the fast analog servo module (FALC110) that executes both slow feedback to the piezoelectric ceramic modulation and fast feedback to the MOD DC current port. These dual feedback mechanisms on the laser controller lead to reductions in the laser linewidth over both short and long time frames.

For the measurement of the laser linewidth, we employ the DSHI method, indicated within the dotted rectangle. The input laser beam is once again divided into two beams. One beam undergoes an 80 MHz shift using an acousto-optic modulator (AOM) to mitigate the influence of low-frequency environmental noise. The other beam is delayed by a single-mode fiber with a specified length of 20.33 km. After combining these two beams with a 1 × 2 fiber coupler, the photocurrent of the beat signal is detected by a fast fiber detector (New Focus, 1554-B). A spectrum analyzer (Rohde & Schwarz, FSVA13) is employed to record the power spectrum of the beat signal.

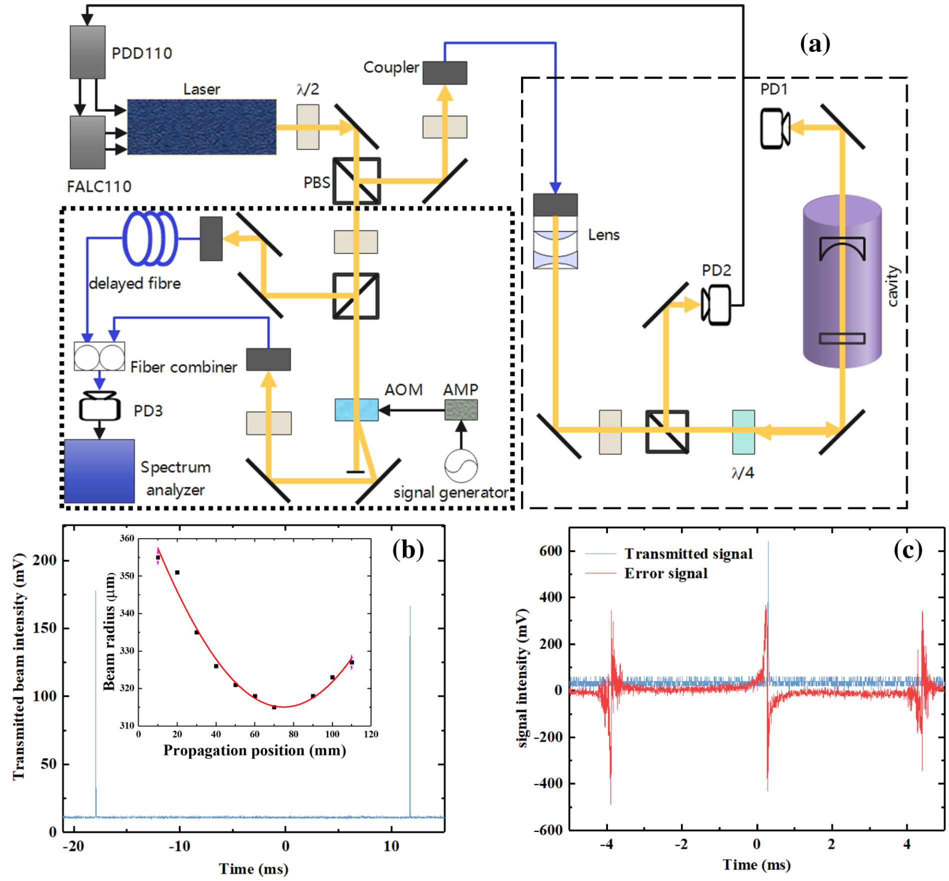

Fig. 1. (a) Experimental setup for obtaining narrow linewidth laser with PDH method (shown in dashed rectangle) and measuring linewidth via the SDSHI method (shown in dotted rectangle). The yellow lines represent optical paths, the blue lines represent optical fibers, and the black lines represent electric cables. (b) Transmitted spectrum measured by PD1 in (a). The two large peaks are the two adjacent modes of optical cavity. The inset shows the measured beam radius around the waist. The red curve shows the fitting results with Gaussian distribution formula. (c) Error signal at one cavity mode position where the laser frequency is locked. $\lambda/2$, half-wave plate; PBS, polarization beam splitter; AMP, power amplifier; AOM, acousto-optic modulator; PD, photodetector.

4. EXPERIMENTAL RESULT

Figure 2 presents our experimental results. The red dots and blue curve in Fig. 2(a) represent, respectively, typical PSD of the beat signal before and after frequency locking using the PDH method. Prior to frequency locking, the optical fiber delay time significantly exceeds the laser's coherence time, resulting in a characteristic Lorentzian profile for the beat signal with a linewidth of approximately 10 kHz. Post-frequency locking, the laser linewidth is considerably reduced, and the fiber delay time is smaller than the coherence time, leading to the emergence of a typical coherence envelope in the SDSHI spectrum, as shown in the enlarged view in Fig. 2(b) for delay fiber length of 20.33 km. The horizontal coordinate values are adjusted to account for the AOM drive frequency (80 MHz) for consistency with the concept explained in Section 1 (Introduction). The spectrum in Fig. 2(b) is normalized relative to the maximum value near the center.

In the subsequent sections, we describe the processing steps of the spectrum presented in Fig. 2(b) based on the proposed approach. The objective is to obtain the ODLS to precisely measure the narrow laser linewidth.

As discussed in Section 2 (Principle), the crux of the demodulated Lorentzian spectrum method lies in determining the periodic modulation signal, S_2, which depends on both the fiber delay time, τ_d, and the linewidth Δf. Although τ_d has been calculated using the formula $\tau_d = nL/c$ in previous studies, inaccuracies in fiber length (L) and the influence of incident light properties (wavelength, polarization) on the refractive index (n) can introduce deviations from the real value. In this context, we derive the value of τ_d from the observed periodic envelope spectrum.

Figure 2(c) displays the positions of all valleys in the SDSHI spectrum shown in Fig. 2(b), plotted as a function of their corresponding order numbers. The linear relationship observed reflects the periodic nature of modulation spectrum S_2. Using the slope obtained from linear fitting, $1/\tau_d = 10037.7 \pm 5.2$ Hz, we calculate τ_d to be 99.624 ± 0.052 μs.

Once τ_d is determined, we can plot the dependence of CDSPSV on the linewidth based on Eq. (6), with both order numbers l and m set to two. The calculated results are shown in Fig. 2(d). CDSPSV is chosen as the metric for estimating the Lorentzian laser linewidth owing to its advantages of low

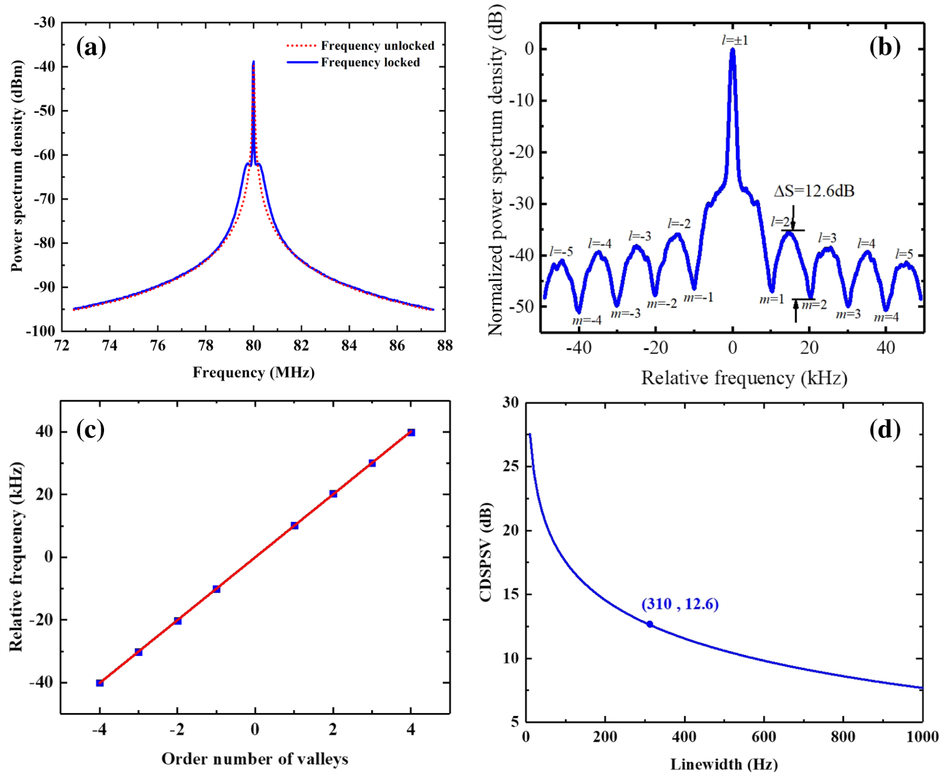

Fig. 2. Short delayed self-heterodyne interferometry spectrum for determining the fiber delay time and estimating the laser linewidth. (a) Power spectrum density of the beat signal with a fiber delay length of 20.33 km when the laser frequency is locked (blue curve) and unlocked (red dots). In the unlocked frequency case, the spectrum exhibits a traditional DSHI Lorentzian profile with a linewidth of approximately 10 kHz. Conversely, under the frequency lock condition, a substructure emerges, featuring a typical coherence envelope of the short DSHI spectrum, as shown in the enlarged view in (b). (c) Positions of all valleys of SDSHI spectrum in (b) as a function of the valley order number for determining the fiber delay time. (d) Dependence of CDSPSV on the linewidth for estimating laser linewidth.

detection error and high stability, with minimal influence from $1/f$ noise and noise floor. For our observed CDSPSV value [12.6 dB, as shown in Fig. 2(b)], the corresponding linewidth is estimated to be 310 Hz.

Following the determination of the fiber delay time (τ_d) and estimated linewidth (Δf_{est}), one can obtain the periodic modulation spectrum and subsequently recover the demodulated Lorentzian spectrum using Eq. (7). In previous studies [20–22], an iterative algorithm was employed to derive the linewidth, with convergence based on the consistency between Δf_{est} and Δf_{upd}. To avoid demodulation errors resulting from the random nature of the initial laser linewidth and repeated iterative calculations, we change the criterion to the fitting of the derived CEDM spectrum with the Lorentzian formula itself.

We generate a series of linewidth values around Δf_{est} to obtain a periodic modulation spectrum (black dotted line) and subsequently demodulated Lorentzian spectrum (blue solid curve) with Lorentzian fitting (red curve), as partially shown in Figs. 3(a) and 3(b) and Fig. 3(d). To compare the fitting effect under different estimated linewidths, we also plot the residual value between the demodulated spectrum and Lorentzian fitting (green curve). When the estimated linewidth is too small, the demodulated Lorentzian spectrum exhibits upward spikes in both wings [Fig. 3(a)], whereas it shows downward spikes when the estimated linewidth is too large [Fig. 3(b)]. To determine the most suitable value, we quantify the degree of Lorentzian fitting by plotting the coefficients of determination as a function of the estimated linewidth [Fig. 3(c)]. The coefficient of determination reflects the agreement between the demodulated Lorentzian spectrum and the standard Lorentzian formula; a higher value indicates a closer fit. A maximum value is noted, indicating the best Lorentzian fitting. We use a polynomial function to fit this relationship, and the optimal Lorentzian spectrum corresponds to a linewidth of 283 Hz, representing the laser linewidth to be measured. It is noted that there are still two small peaks left on both sides of the center frequency for the optimal demodulated spectrum [labeled with red shadows in Fig. 3(d)]. In fact they exist in the original power spectrum in Fig. 2(b) and are supposed to arise from laser intensity noise, which is also observed in [23].

To verify the correctness and reliability of our present method, we change the delay fiber length and record corresponding coherent envelope spectra. Figure 4(a) shows three typical coherent envelope spectra with delay fiber lengths of 20.33 and 35.33 km. Each spectrum is an averaged result of 100 times. We use the ODLS method we present to derive linewidths and plot the dependence on delay fiber length, along with the values derived with CDSPSV and with an iterative algorithm. Each dot is the averaged value of three measurements and the error bar is the corresponding standard deviation. There is a lack of data for the iterative algorithm method at a delay fiber length of 5 km due to that the derived linewidth is not

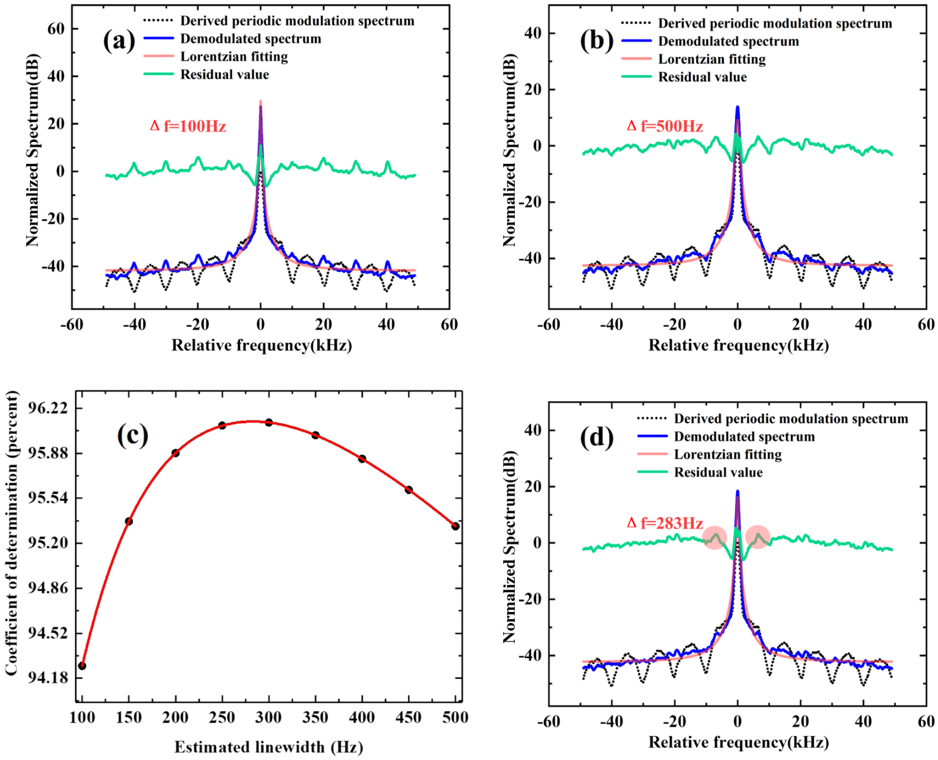

Fig. 3. Measurement of narrow laser linewidth with the optimal demodulated Lorentzian spectrum. (a), (b), (d) Demodulated Lorentzian spectra (blue curves) with estimated laser linewidths of 100, 500, and 283 Hz, respectively. The black dots show the corresponding modulation signal, and the red curves indicate Lorentzian fitting. (c) Coefficient of determination for the demodulated Lorentzian spectrum as a function of the estimated linewidth, with the red curve representing the experimental fitting using a polynomial function.

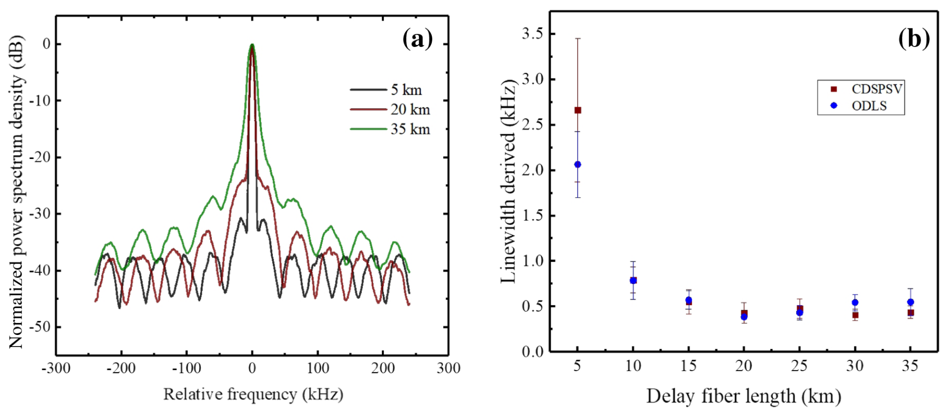

Fig. 4. Dependence of (a) coherent envelope spectra and (b) derived linewidth on delay fiber length.

converged. It shows that the linewidths derived with our present ODLS are more coincident with the CDSPSV method than with the iterative algorithm method. It is expected that the laser linewidths under locking status have little change when delay fiber length varies. Thus, the CDSPSV and ODLS methods look better than the iterative algorithm because the fluctuations for them are little when fiber length is above 15 km, while the values derived with the iterative algorithm cover a much larger range. The reason why the value below the fiber length of 10 km strongly deviates from other derived values (the value with our present method deviates relatively weakly) is that the influence from the noise floor on coherent envelope spectra near the center frequency increases when the delay fiber length decreases. A similar influence is also observed in [16].

5. CONCLUSION

We introduced an ODLS method based on the SDSHI spectrum to accurately measure narrow laser linewidths. This measurement is accomplished by locking a semiconductor laser to a high-finesse ultra-stable optical cavity using the PDH method. Our approach begins with the coherence envelope of the SDSHI spectrum, from which we derive the fiber delay time by leveraging the periodic characteristics of the envelope spectrum. This approach avoids deviations from real values that may occur with traditional methods due to inaccuracies in fiber length and refractive index determination. Based on the determined fiber delay time, we calculate the dependence of CDSPSV on the linewidth. In typical ADCCE methods, the linewidth corresponding to the observed CDSPSV is often considered the final measured value. However, in reality, the actual linewidth is expected to be larger. To obtain the true linewidth accurately, we recover a CEDM spectrum by dividing the periodic modulation spectrum with the observed SDSHI spectrum and fit it with a Lorentzian formula. Previous methods typically employed iterative algorithms to derive the linewidth, relying on the consistency between the estimated and updated linewidth values. This approach was susceptible to demodulation errors due to the random characteristics of the initial laser linewidth and repeated iterations. In contrast, we propose a new approach to obtain the CEDM spectrum, with the criterion being the best fit of the derived CEDM spectrum with the Lorentzian formula. We quantify the degree of Lorentzian fitting using coefficients of determination obtained from the demodulated spectrum. Remarkably, we discover the existence of a maximum value by plotting the coefficients of determination against a range of linewidth values around the estimated figure. Consequently, we obtain the optimal demodulated Lorentzian spectrum and the corresponding linewidth, which is supposed to be the closest approximation to the real linewidth. Finally, we verify the correctness of our present ODLS method with a comparison with the CDSPSV method, but need to avoid the influence from the noise floor.

This measurement method for laser linewidth offers several advantages. It eliminates Gaussian broadening caused by $1/f$ noise compared to the traditional long fiber DSHI method. Additionally, it avoids errors introduced by manually selecting data points, a common issue in the ADCCE method. By not relying on random initial laser linewidth estimates or repeated iterative algorithms, as seen in previous approaches, we obtain the optimal demodulated Lorentzian spectrum and the corresponding laser linewidth more objectively. Therefore, our proposed method holds significant importance for the measurement of laser linewidth, particularly for narrow and ultranarrow laser linewidths.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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